Theoretical Calculations Of The Transmission Probability In Mesoscopic Systems

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***Abstract*—In this paper, we present a simple theoretical description of recently measured transmission coefficient in two-dimensional (2D) saddle-point potential. Our study is at absence and presence of a perpendicular magnetic field. A simple analytical expression obtained for the transmission coefficient through the saddle point. Our results showed good agreement with the calculations of the transmission coefficient in the WKB approximation. In addition, our analysis makes use of the fact that, for large values of the distance of the closest approach of the electron to the saddle point and Hamiltonian for this system can express as a sum of two commuting Hamiltonians. One of them is involving only the cyclotron coordinates, and the other one involving only the guiding-center coordinates. The previous has the form of a 1-D particle in a confining harmonic potential and describes the vibrations of the electron in guiding-center position.**

Keywords—Magnetic field, Transmission, Saddle-point, Conductance.

INTRODUCTION

Saddle points are the points on the potential energy surface where the gradient is zero. In the most general terms, a saddle point for a smooth function is a stationary point such that the curve/surface and other. In the neighborhood of a saddle point is not entirely on any side of the tangent space at that point. In dynamical systems, a saddle point is a periodic point that's stable and unstable manifolds have a dimension that is non-zero.

If a differentiable map f gives the dynamic, then a point is hyperbolic if and only if the differential of f^n (n is the period) has no eigenvalue on the unit circle in case of a complex when computed at the point. In this paper, we will follow the theoretical discussion discovered by V.Wees [1] and Wharam [2] in split-gate constrictions of a 2D electron gas [3]. The literature [4-16] treats this problem by considering a hard-wall potential. Some problems considering the width of the conduction channel are also unspecified to change abruptly, but in the other problems [13-16] it is assumed to be a smooth function. As is well known, the transmission coefficient represents the probability flux of the transmitted wave about that of the incident wave. In addition, the quantum-mechanical solution of this scattering problem is simple [17, 18] and permits physical insight. A complete discussion of constriction conductance requires the consideration of carrier transmission from one equilibrium electron reservoir to another. However, if the transmission is globally adiabatic [5, 13-16], the calculation of the conductance due to the local scattering at the saddle is accurate up to exponentially small corrections [16].

2. Analysis and discussion

Near the bottleneck of the constriction the electrostatic potential can be expanded, and regarding appropriate coordinates, x and y is given by

$$V_{sp}(x,y)=\frac{1}{2}mw_y^2y^2-\frac{1}{2}mw_x^2x^2+V(x,t). \qquad (1)$$

Here, $V(x,y)=V_{\max}\sin(wt)$ is the electrostatic potential at the saddle, and the curvatures of the potential are expressed regarding the frequencies w_x *and* w_y and we can neglect the higher-order terms in x and y. Let us first study the case of zero magnetic fields. The total energy is the sum of kinetic energy $p^2/2m$ supplemented by potential energy as given by equation (1).

We obtain the result;

$H = H_\circ + H_1$, where the Hamiltonian $H_\circ$ is a transverse wave function associated with energy $\hbar w_y\left(n+\frac{1}{2}\right)$, n=0,1,2… and H_1 is a wave function for motion along x in an effective potential $V(x,t)+\hbar\omega_y\left(n+\frac{1}{2}\right)-\frac{1}{2}m\omega_x^2x^2$. In a saddle point region can be viewed effective potential as the band bottom of the nth quantum channel (sub-band) [13]. In the absence of quantum tunneling (i.e $E_n \langle\langle$ V(x,t)) the channels with threshold energy given by.

$$E_n=\hbar w_y\left(n+\frac{1}{2}\right)+V_{\max}\sin(wt). \qquad (2)$$

The channels with threshold energy E_n above the Fermi energy are closed; the other channels below the Fermi energy are open. Quantum mechanically transmission and reflection at the saddle allows for channels which are neither completely open nor completely closed, but which permit transmission with a probability T_{mn}. Here, the index n refers to the incident channel and the index m refers to the outgoing channel. The transmission probability is calculated in [17] and it can be written in the form.

$$T = \frac{1}{1+e^{-\pi\varepsilon_n}}, \quad (3)$$

where

$$\varepsilon_n = \frac{2\left(E-\hbar w_y\left(n+\frac{1}{2}\right)-V_{max}sin(wt)\right)}{\hbar w_x}.$$

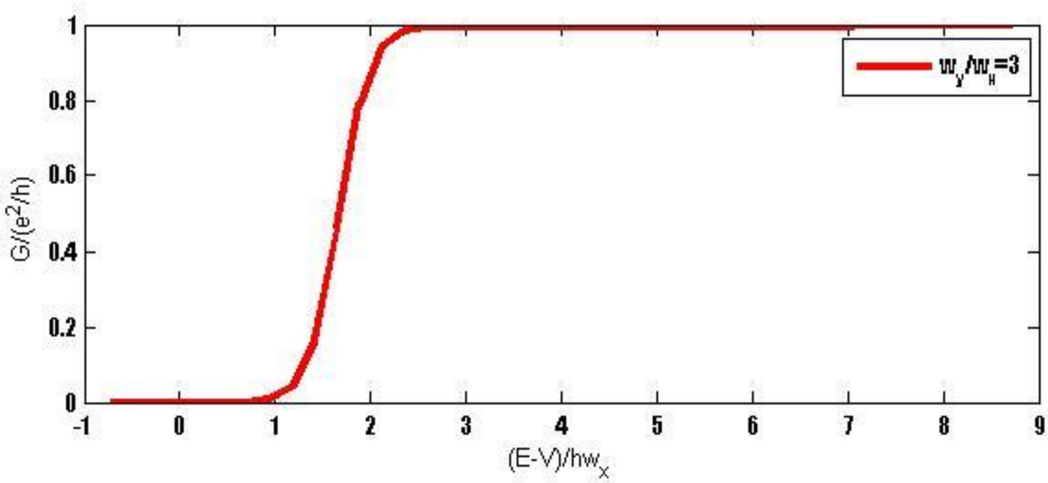

Fig.1: The total transmission probability (conductance) as a function of (E-V)/hw_x for a ratio of w_y/w_x=3. The opening of successive quantum channels over narrow energy intervals leads to the quantization of the conductance.

From Fig.1 we can note that, for $\varepsilon_n \gg 0$ the transmission probability is close to one, $T \approx 1 - \exp(-\pi\varepsilon_n)$. Also the transition from zero transmission probability to a transmission probability close to one occurs near $\varepsilon_n = 0$ i.e., in the neighborhood of the classical threshold energy E_n given by equation (2). The size of the energy interval needed for the transition is determined by the term $\hbar w_x$. We can determine the conductance according to the value of transmission from one equilibrium electron reservoir to another equilibrium reservoir by the sum of all transmission probabilities,$T = \sum_{mn} T_{mn,}$ and is given by $G = \frac{e^2}{h}T$ [3, 4, 19, and 20].

3. Exact Calculation of the transmission coefficient.

Let us consider a constriction formed by a saddle-point potential in 2D $V_{sp}(x,y)=\frac{1}{2}mw_y^2y^2-\frac{1}{2}mw_x^2x^2+V(x,t)$ in a uniform magnetic field B (perpendicular to the x and y plane), and in the symmetric gauge, the vector potential is given by A= (B/2) (-y, x, 0). Cyclotron motion with frequency $\omega_c=\frac{eB}{mc}$ gives rise to a new energy scale which affects the transmission behavior. We can obtain the transmission probabilities have the following form [18].

$$T_{mn}=\sum\left(1+\exp\left[-2\pi\left(E-V_{\max}\sin(wt)-E_2\left(n+\frac{1}{2}\right)\right)\Big/E_1\right]\right)^{-1}. \quad (4)$$

Where

$$E_1=\left[\gamma^2-\left\{\frac{1}{2}\Omega-\left(\gamma^2+\left(\frac{w_c}{2}\right)^2\right)^{1/2}\right\}^2\right]^{1/2}, \quad (5)$$

$$E_2=\left[\left(\frac{\Omega}{2}+\left(\gamma^2+\left(\frac{w_c}{2}\right)^2\right)^{1/2}\right)^2-\gamma^2\right]^{1/2}, \quad (6)$$

and

$$u_-=w_y^2-w_x^2,\ u_+=w_y^2+w_x^2,\quad \gamma=\frac{u_+}{\Omega},\ \Omega=\left(\frac{w_c^2}{4}+\frac{u_-}{2}\right)^{1/2}.$$

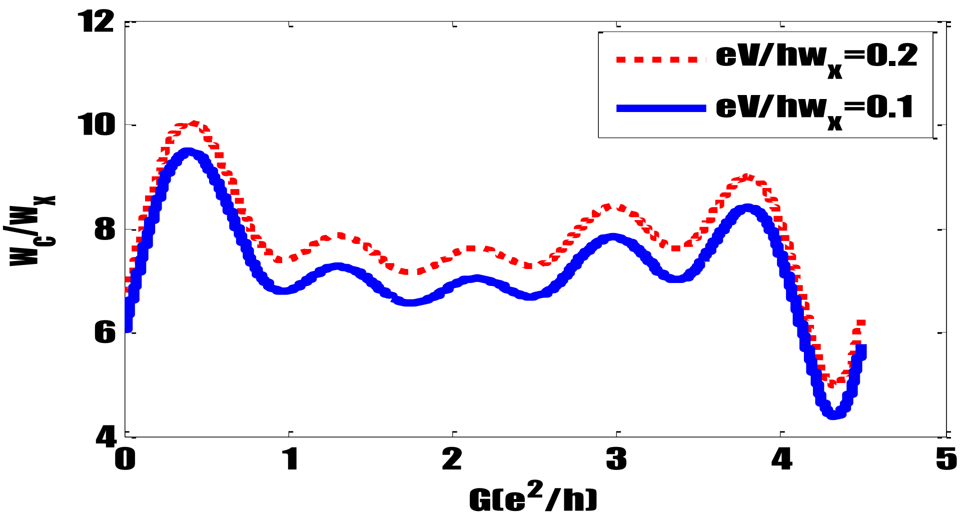

Fig.2: The conductance (G, in units of e^2/h) of a 2D contact plotted vs the dimensionless cyclotron frequency w_c/w_x with $w_y/w_x=3$. Different curves correspond to the marked values of the applied voltage (V in units of hw_x/e).

We recognize that energy levels for systems with an orbital angular momentum that were degenerate outside of an applied magnetic field, with degeneracy of have the same degeneracy lifted inside the field. Thus, for example, a system with this type of splitting is not observed because of coupling between orbital angular momentum and the intrinsic angular momentum of the electron suggested by Uhlenbeck and Goudsmit.

However, we can use the prediction of the behavior of a system with electronic orbital angular momentum in a magnetic field to more clearly understand the magnitude of the intrinsic angular momentum, or spin. Therefore, under the effect of magnetic field, the transverse energy levels of the electrons equal to $\hbar E_2\left(n+\frac{1}{2}\right)$. The magnetic field shifts the transverse energy levels due to the appearance of an vibrating structure of the transmission probabilities. In Fig.2 for several values of V there is a dependence of the constriction conductance on the dimensionless cyclotron frequency w_c/w_x for a contact with w_y/w_x=3. The increment of the voltage leads to differences in the behavior of the conductance (upper curves). The behavior of the conductance can be influenced either by an applied magnetic field, applied voltage, or combinations of the both.

4. Semi classical Quantization for the motion of Guiding Center:

The charged particles are perpendicular to both the direction of motion and the magnetic field line. It depends on much energy the particles have as well as the strength of the magnetic field. However, if the magnetic field is strong enough, and the particle loses a little energy, by occasionally colliding with other particles, it can get trapped by the magnetic field, at which point it will circle it. The particles may tend to spiral around magnetic field lines, movable along the lines as they orbit nearly them. That part of their velocities that is punctuation along the magnetic field is not affected, but the part that's perpendicular circles them. The physics of charged particles in the high magnetic field has been one of the significance problems in quantum mechanics insufflated by condensed matter physics. The unique feature is that the system described by two degrees of freedom, namely, the motion of cyclotron and guiding center if one chooses the particular type of gauge for writing the vector potential. In the case that we have the uniform magnetic field only, the guiding center degrees of freedoms is not active; this manifested by the infinite degeneracy of Landau levels. The degeneracy implies some symmetry which is governed only by the guiding center coordinates. Now if a potential of non-magnetic origin added, the guiding center degrees of freedom begins to be active. The energy exchange occurs between the motions of the cyclotron and guiding center, namely, we expect the mixing between inter-Landau levels. We can focus our attention on the motion of the guiding center in the high magnetic field, and by some mathematical analysis we can get the form of transmission probability:

$$T_{mn} = \frac{1}{1+\exp(-\pi\varepsilon)} \qquad (7)$$

Where ε is a dimensionless measure of the energy of the guiding – center motion relative to the potential $V_{\max}\sin(wt)$ at the saddle point by this form:

$$\varepsilon = {\left(E_G - V_{\max} \sin(wt)\right)}\Big/{E_1}, \qquad (8)$$

$$E_1 = \left(\left(\frac{U_+^2}{8m^2} + \left(\frac{w_c \Omega}{4} \right)^2 \right)^{1/2} - \frac{w_c^2}{2} - \frac{U_-^2}{m^2} \right)^{1/2}.$$

(9)

Where E_G is the energy of the guiding center motion. If the electron move in a pure state with quantum number n within its oscillations about the guiding center, then $E_G = E - \left(n + \frac{1}{2}\right)E_2$, where E is the total energy of the electron, and E_2 is the oscillator frequency,

$$E_2 = \left(\left(\frac{U_+^2}{2m^2} + \left(\frac{w_c \Omega}{4} \right)^2 \right)^{1/2} + \frac{w_c^2}{2} + \frac{w_y^2 - w_x^2}{m} \right)^{1/2}.$$

(10)

The transition amplitude of one-dimensional (1-D) particle passing through an inverted parabolic barrier has been studied by Connor [17] in the context of resonance tunneling reactions. Although the results of Connor [17] are closely related to our results, however, he does not solve explicitly for the transmission coefficient of this system. Therefore, we obtained few details of the calculation below to find $T_{1D}= (E_G - V_0)$. We have solved explicitly H1 Ψ(x) = (E_G-V_0) Ψ(x). Writing p= (1/i) (d/dx) and ε= (E_G-V_0)/E_1, The equation become

$$\left(\frac{d^2}{dx^2} + w_x^2 x^2 + \varepsilon \right) \Psi(x) = 0.$$

(11)

Equation (11) is discussed in details by Norse and Feshbach [22]. The solutions are called parabolic cylindrical functions. For each value of ε, there is an even and odd solution, which we denote respectively as $\Psi_e(x)$ and $\Psi_0(x)$. These solutions may be expressed in terms of confluent hypergeometric functions $F(a\,|\,b\,|u)$ as

$$\Psi_e(x) = e^{-iw_x x^2/2} F\left(\frac{1}{4} + \frac{1}{4w_x} i\varepsilon \,\middle|\, \frac{1}{2} \,\middle|\, iw_x x^2 \right) \qquad (12)$$

$$\Psi_0(x) = x\sqrt{w_x}\, e^{-iw_x x^2/2} F\left(\frac{3}{4} + \frac{1}{4w_x} i\varepsilon \,\middle|\, \frac{3}{2} \,\middle|\, iw_x x^2 \right) \qquad (13)$$

We now examine the solutions to Eq.(11) for large values of $|\,x\,|$, writing $u = |\,u\,| e^{i\theta}$, one can show:

$$F = (a\,|\,b\,|u) \rightarrow |\,u\,|^{a-b} e^{i(a-b)\theta} e^{u} \frac{\Gamma(b)}{\Gamma(a)} + |\,u\,|^{-a} e^{ia(\pi-\theta)} \frac{\Gamma(b)}{\Gamma(b-a)}, \qquad (14)$$

$$\Psi_e(x) \to e^{-iw_x x^2/2} \left[\frac{\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{1}{4}+\frac{i\varepsilon}{4w_x}\right)} \exp\left(-i\left(\frac{1}{4}-\frac{i\varepsilon}{4w_x}\right)\frac{\pi}{2}\right) |x|^{-2\left[\frac{1}{4}-\frac{i\varepsilon}{4w_x}\right]} e^{ix^2} + \frac{\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{1}{4}-\frac{i\varepsilon}{4w_x}\right)} \exp\left(i\left(\frac{1}{4}+\frac{i\varepsilon}{4w_x}\right)\frac{\pi}{2}\right) |x|^{-2\left[\frac{1}{4}-\frac{i\varepsilon}{4w_x}\right]} \right], \qquad (15)$$

$$\Psi_0(x) \to x\sqrt{w_x}\, e^{-iw_x x^2/2} \left[\frac{\Gamma\left(\frac{3}{2}\right)}{\Gamma\left(\frac{3}{4}+\frac{i\varepsilon}{4w_x}\right)} \exp\left(-i\left(\frac{3}{4}-\frac{i\varepsilon}{4w_x}\right)\frac{\pi}{2}\right) |x|^{-2\left[\frac{3}{4}-\frac{i\varepsilon}{4w_x}\right]} e^{ix^2} + \frac{\Gamma\left(\frac{3}{2}\right)}{\Gamma\left(\frac{3}{4}-\frac{i\varepsilon}{4w_x}\right)} \exp\left(i\left(\frac{3}{4}+\frac{i\varepsilon}{4w_x}\right)\frac{\pi}{2}\right) |x|^{-2\left[\frac{3}{4}-\frac{i\varepsilon}{4w_x}\right]} \right]. \qquad (16)$$

We see that there is one term in the equation of Ψ_0 and the equation of Ψ_e is proportional to $\exp(iw_x x^2/2)$ as well as proportional to $\exp(-iw_x x^2/2)$ for large values of $|x|$. We will try to associate one of these terms with the incoming current and the other with the outgoing current. Noting that $pe^{\pm iw_x x^2/2} = \pm xe^{\pm iw_x x^2/2}$, we see that in the current associated with another term proportional to $e^{iw_x x^2/2}$ is directed away from the origin. The current associated with the term proportional to $e^{-iw_x x^2/2}$ is directed toward the origin. We thus associate the former with the outgoing current, and the latter with the incoming current. We proceed by forming an eigenstate of the form $\Psi(x) = A\Psi_e(x) + B\Psi_0(x)$, where the coefficients A and B are chosen such that for large positive values of x the coefficient of the $e^{-iw_x x^2/2}$ term vanishes. The physical picture of this situation can be viewed as, on the right of x=0 there is only an outgoing current, while on the left side of the origin there is both an incoming and an outgoing current. For large values of $|x|$ we denote the asymptotic forms of the wave function associated with the incoming and outgoing current as $\Psi_{in}(u)$ and $\Psi_{out}(u)$, respectively. The transmission coefficient may be written as.

$$T_{1D} = \lim_{x\to\infty} \frac{|\Psi_{out}(x)|^2}{|\Psi_{in}(-x)|^2} . \qquad (17)$$

In accordance with the above discussion, we now choose A and B to satisfy the equation.

$$A\frac{\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{1}{4}-\frac{i\varepsilon}{4w_x}\right)}e^{i\pi/8}+B\frac{\Gamma\left(\frac{3}{2}\right)}{\Gamma\left(\frac{3}{4}-\frac{i\varepsilon}{4w_x}\right)}e^{i3\pi/8}=0. \quad (18)$$

for large values of $x\rangle\, 0$, one finds $\Psi(x)\rightarrow\Psi_{out}(x)$, with

$$\Psi_{out}(x)=|x|^{-2[(1/4)+(1/4)i\varepsilon]}e^{-\pi\varepsilon/8}e^{ix^2/2}\begin{pmatrix}A\left(\Gamma\left(\frac{1}{2}\right)\Big/\Gamma\left(\frac{1}{4}+\frac{i\varepsilon}{4w_x}\right)\right)e^{-i\pi/8}\\ +B\left(\Gamma\left(\frac{3}{2}\right)\Big/\Gamma\left(\frac{3}{4}+\frac{i\varepsilon}{4w_x}\right)\right)e^{-i3\pi/8}\end{pmatrix}. \quad (19)$$

For large negative x, one finds.

$$\Psi in(x)=|x|^{-2[(1/4)+(1/4)i\varepsilon]}e^{-\pi\varepsilon/8}e^{-ix^2/2}\begin{bmatrix}A\left(\Gamma\left(\frac{1}{2}\right)\Big/\Gamma\left(\frac{1}{4}-\frac{i\varepsilon}{4w_x}\right)\right)e^{i\pi/8}\\ -B\Gamma\left(\frac{3}{2}\right)\Big/\Gamma\left(\frac{3}{4}-\frac{i\varepsilon}{4w_x}\right)e^{i3\pi/8}\end{bmatrix}, \quad (20)$$

substituting Eq. (18) into Eqs. (19) and (20), and using Eq. (17), we find

$$T_{1D}=\frac{1}{4}\left|\frac{\Gamma\left(\frac{1}{4}-\frac{i\varepsilon}{4w_x}\right)}{\Gamma\left(\frac{1}{4}+\frac{i\varepsilon}{4w_x}\right)}e^{i\pi/4}-\frac{\Gamma\left(\frac{3}{4}-\frac{i\varepsilon}{4w_x}\right)}{\Gamma\left(\frac{3}{4}+\frac{i\varepsilon}{4w_x}\right)}e^{-i\pi/4}\right|^2. \quad (21)$$

This form can be greatly simplified using the identities $\Gamma(x+iy)=\Gamma^*(x-iy)$,

$$and\quad \Gamma\left(\frac{1}{4}+iy\right)\Gamma\left(\frac{3}{4}-iy\right)=\frac{\pi\sqrt{2}}{cash(\pi y)+i\sinh(\pi y)},$$

where x and y are arbitrary real numbers[21]. We find

$$T(E_G)=T_{1D}(E_G-V_\circ)=\frac{1}{1+\exp(-\pi\varepsilon)}. \quad (22)$$

Therefore, in the presence of the field, E_2 takes the plays of $\hbar w_y$ and E_1 plays the role $\hbar w_x/2$ plays at zero fields. Again, there is no channel mixing. In the limit of zero applied field, these formulas reduce to the results which is presented above. At high fields, when w_c exceeds w_x and w_y Eqs.(9) and (10) simplify considerably. The relevant energies are [18].

$$E_1 \approx \left(\hbar\omega_x\omega_y/2\omega_c\right) \approx (m/2)\omega_x\omega_y l_B^2 . \qquad (23)$$

where we have used the magnetic length $l_B^2 = \hbar c/_{|eB|}$ and

$$E_2 \approx \hbar\omega_c . \qquad (24)$$

Where Eqs. (23) and (24) are applicable, a simple interpretation is again possible. In high magnetic field the carriers execute rapid cyclotron motion around a guiding center with energy $E_G = E - \hbar w_c\left(n + \frac{1}{2}\right)$, the equipotential contours of the potential in Eq. (1). In the absence of tunneling, the trajectories of the states in a high magnetic field are determined by E_G=V(x,y). For E_G 〈 V(x,t) this describes a trajectory which is repelled by the saddle. The closest approach of such a trajectory to the saddle point is determined by E_G =V(x_n ,0). For E_G 〉 V(x,t) the trajectory passes through the saddle. The closest approach of such trajectory to the saddle-point is determined by E_G=V(0,y_n) as has been pointed out in Ref.[18], if $\varepsilon_n < 0$, the transmission probabilities in terms of x_n or y_n are in the high- magnetic-field limit.

$$T_{mn} = \delta_{mn}\frac{1}{1+\exp\left[\pi(\omega_x/\omega_y)(x_n/l_B)^2\right]},$$
(25)

$$T_{mn} = \delta_{mn}\frac{1}{1+\exp\left[-\pi(\omega_x/\omega_y)(x_n/l_B)^2\right]}$$
. (26)

If $\varepsilon_n > 0$ since $((x_{n+1}^2 - x_n^2)/l_B^2 = 2w_c^2/w_x^2 \gg 1$, it is at most one of the transmission probabilities which is between zero and one (up to exponentially small corrections). All the high-magnetic field states, except possibly one, are either completely reflected at the saddle or completely transmitted [1].

Notice that this implicitly assumes that the classical motion of the electron is confined either to the left or the right of the saddle point; for x_n 〉〉 l_B we have

$$T_{mn} = \exp\left[-\pi(\omega_x/\omega_y)(x_n/l_B)^2\right]. \qquad (27)$$

4. 5. The Connection Formula: As it well known, the WKB method is a method for finding approximate solutions to linear partial differential equations with varying coefficients. In quantum mechanics, the wave function is assumed an exponential function with amplitude and a phase that slowly varies compared to the de Broglie wavelength, λ, and is semi-classically. The heart of any discussion of WKB methods is the appropriate connection formula. Our definition of the transmission coefficient, T, is such that it corresponds to quantum tunneling through the saddle-point for energies E_G〈 V(x,t),while it corresponds to the classical motion around the saddle-point for positive energies E_G〉V(x,t). Note that the transmission in the x direction is equivalent to the reflection in the y direction, Consider a potential barrier and energy such that EG〉V(x,t). The WKB solutions to the left and right of the barrier are similar. This result in Eq. (27) may obtained for ε <0 by applying the WKB approximation in the Hamiltonian H_1.

$$\Psi_I = \frac{A}{[k(x)]^{1/2}} e^{i\omega_1(x)} + \frac{B}{[k(x)]^{1/2}} e^{-i\omega_1(x)},$$
(28)

$$\Psi_{II} = \frac{C}{[k(x)]^{1/2}} e^{i\omega_2(x)} + \frac{D}{[k(x)]^{1/2}} e^{-i\omega_2(x)}$$
(29)

where

$$k(x) = \left[2m(\varepsilon - V(x))/\hbar^2\right]^{1/2}.$$
(30)

The connection formula relating the constants A and B to C and D is given by Froman [21] as

$$T_{WkB} = \left(\frac{4Z}{1+4e^{2Z}}\right)^2,$$
(31)

where the usual barrier penetration integral is:

$$Z = \int_{-x}^{x} |k(x)| dx . \qquad (32)$$

We wish to expand $\mathbb{Z}$ in powers of the energy relative to the potential maximum. For definiteness, we suppose that E 〈 V $_{max}$, but the result is also valid for E 〉V $_{max}$. Let $x_\circ$ by the position of the maximum of V(x) and the expansion parameter is λ.

Now we have that $\mathbb{Z}$ is given by

$$Z = \left(\frac{2m}{\hbar^2}\right)^{1/2} \int_{-x_\circ}^{x_\circ} [V-\varepsilon]^{1/2} dx \equiv \exp\left(\frac{\pi x^2}{2}\right),$$
(33)

where $x_\circ^2 = -\varepsilon$

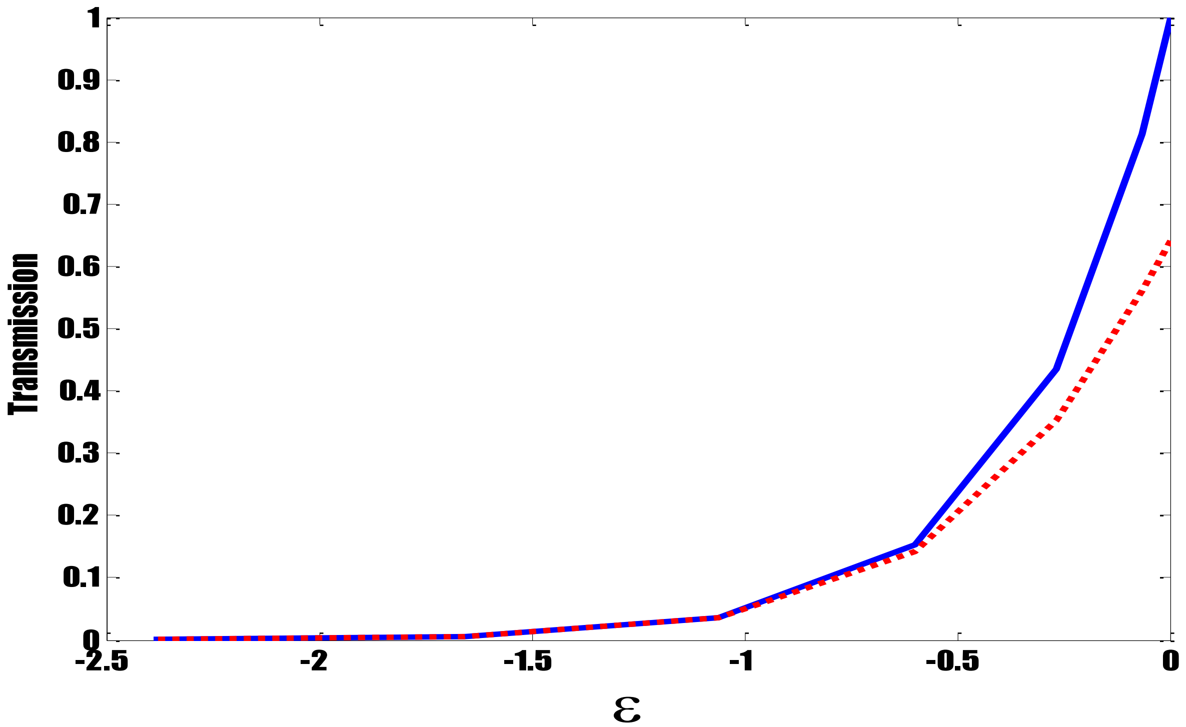

Fig.3: Transmission coefficient of an electron through a saddle-point potential in a strong magnetic field as a function of the dimensionless parameter ε defined by Eqs.(10,11). Dashed line shows WKB approximation of the transmission.

In the strong field limit, we can obtain the distance of the turning points from the origin in the 1-D problem ($x_\circ$) corresponds to the distance of closest approach of the electron to the saddle point ($x_\circ$) in the 2D problem. We see that in the limit $|\varepsilon| \gg 1$, T_{WKB} is approximately $\mathrm{Exp}(-\pi|\varepsilon|)$. It follows that regarding to $x_\circ$,T_{WKB} has precisely the same form as T in Eq. (31) for $^{x_\circ}/_{l_B} \gg 1$ in the strong magnetic field limit. In Fig. (3), we plot T_{WKB} with the exact transmission coefficient T in the strong-field limit as a function of the parameter, ε. For $\varepsilon \gg 1$, T_{WKB} and T agree quite well. As $\varepsilon \to 0$, we find T_{WKB}= (16/25). In contrast, the exact transmission coefficient equals 1 for $\varepsilon = 0$. By the symmetry considerations, WKB approximation breaks down for small values of $|\varepsilon|$ is not surprising. May this case represent the situation in which the classical turning points of the 1-D problem are close to one another. The WKB approximation often does poorly when this occurs. Although the approximations in the WKB method do poorly for small values of $x_\circ$, we expect that for large values of $x_\circ$ it should give us a good representation of the exact transmission coefficient. This gives us a limit in which we check the correctness of our exact evaluation of T. As we see in Fig. (3), the agreement in this limit is quite good. Also, we will use these results to solve other problems by details and connect with our new results [23-26].

6. Conclusion

During this paper, we have calculated the transmission coefficient T for an electron in an arbitrary magnetic field and a saddle-point. The following conclusions can be drawn from the results:

1- We have obtained a simple theoretical description to transmission coefficient in 2Dl saddle-point potential in the two cases; absence and presence of a perpendicular magnetic field.

2- The results agree well with the WKB approximation. In our exact calculation, we have expressed the Hamiltonian as a sum of the two Hamiltonians, one involving only the cyclotron coordinates s and p, the other involving only the guiding-center. The guiding-center Hamiltonian is that of a 1-D particle in an inverted harmonic.

3- The motion of the wave packet is consistent with the semi classical picture of the Eigen functions of an electron in a strong magnetic field and a slowly varying potential. The probability that the particle in 2-D will tunnel through the saddle-point barrier is equivalent to the probability that the 1-D particle will be transmitted through the inverted harmonic-oscillator potential.

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