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Analytical Modeling of a DC Motor Driven Pump Sprinkler Irrigation Nozzle and Plant Water Uptake Process for Application in Automatic Irrigation Process

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***Abstract*—In this paper, analytical modeling of a DC motor driven pump sprinkler irrigation nozzle and plant water uptake process for application in automatic irrigation process is presented. For the DC motor driven irrigation water pump the transfer function for the pump-motor relationship that yield voltage as input to the system and pump flow rate as the output was derived. The analytical model for the nozzle orifice discharge flow rate and the Sprinkler head were also derived along with the expressions for the gross application depth of the sprinkler, the system capacity and the plant water uptake. The models and associated ideas presented in this paper are relevant for the simulation and parametric analysis of the DC motor driven pump sprinkler irrigation nozzle and plant water uptake process for automatic irrigation process.**

Keywords—Irrigation, Water Uptake Process, Sprinkler Irrigation, DC motor, flow rate, sprinkler head

I. Introduction

All over the world today, there has been a growing need to ensure food security [1,2,3]. One way of achieving this is improving crop yield and production through improved mechanized farming system [4,5,6]. Notably, irrigation has also been identified as a viable means of improving crop yield [7,8,9,10]. Irrigation is defined as the artificial application of water, with good economic returns and no damage to soil, to supplement the natural sources of water to meet the water requirement of plants [11,12].

Irrigation is one of the long standing practices of agricultural engineering which is used to assist in the growing of agricultural crops, maintenance of landscapes and re-vegetation of disturbed soils in dry areas or during periods of inadequate rainfall [13]. Additionally, improving irrigation efficiency can contribute greatly to reducing production costs of vegetables, making the industry more competitive and sustainable [14].

In any case, the problem of the determination of the adequate amount and the correct time at which irrigation water should be applied to plants has always been a major issue. Therefore, effective design of a real-time automatic irrigation system requires that an accurate analytical model should be derived to enable the simulation of the system. The analytical models will also make it possible carry out control analysis of the system which largely depends on the development of accurate soil moisture content model , as well as the plant water uptake model. As such, in this paper, the focus is to develop analytical models for soil moisture content and plant water uptake based on Richard's equation. Also, the paper will seek to develop analytical models for a DC motor driven centrifugal pump, irrigation sprinkler nozzle , sprinkler application rate, precipitation rate and gross depth, the irrigation system capacity and the plant water uptake process. Eventually, the relevant transfer functions that will enable the models to be applied in the design of intelligent control systems for efficient automated irrigation process are derived.

II. Methodology

A. Modeling of a DC Motor Driven Irrigation Pump

In this paper, the sprinkler irrigation system for the farm gets its water from a reservoir. The reservoir is filled with water by a DC motor driven centrifugal pump as shown in the schematic diagram in Figure 1.

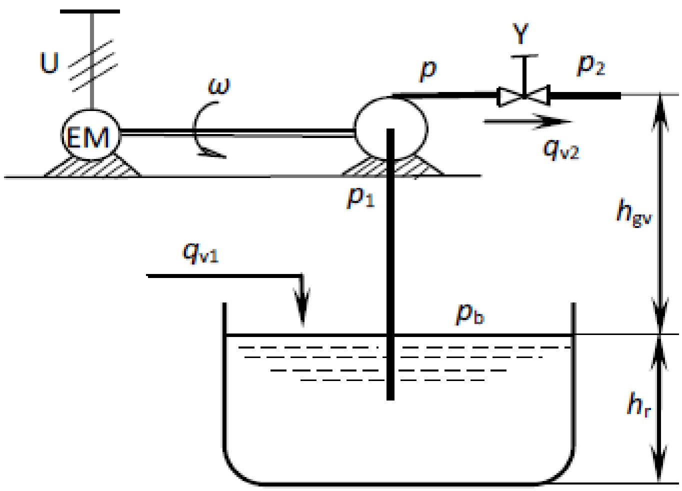

Figure 1: The schematic diagram of the pump-motor system for the sprinkler irrigation

The electrical circuit of the pump-motor system has a voltage source (V_a) across the coil of the armature with an inductance (L) in series with a resistance (R) in series with induced voltage V_e which opposes the voltage source. Using Kirchhoff's voltage law around the electrical loop, the sum of all voltage around the loop is zero;

$$V_a - V_R - V_L - V_e = 0 \qquad (1)$$

Applying Ohm's law, the voltage, $V_R$across the resistor is;

$$V_R = Ri \qquad (2)$$

Where i is the armature current. The inductor terminal voltage is given as;

$$V_L = \frac{Ldi}{dt} \qquad (3)$$

Hence, the induced back emf is written as;

$$V_e = K_b\omega \qquad (4)$$

Where K_b is the *velocity* constant. Substituting Equations (2), (3) and (4) into Equation (1), gives;

$$V_a - Ri - \frac{Ldi}{dt} - K_b\omega = 0 \qquad (5)$$

Solving for the applied voltage ,$V_a$gives;

$$V_a = Ri + \frac{Ldi}{dt} + K_b\omega \qquad (6)$$

Where ω is the angular velocity and K_b is the *velocity* constant.

The dynamic equation of the mechanical part of the motor-pump is based on Newton's law, which states that the resultant torque on the motor shaft must be equal to zero, so;

$$\tau - T_r - T_v - T_p = 0 \qquad (7)$$

where τ is the electromagnetic torque, T_r is the torque due to rotational acceleration of the rotor, T_v is torque associated with velocity of rotor, and T_p is the torque of the pump impeller. Now, τ, T_r and T_v are given as follows;

$$\tau = K_m i \qquad (8)$$

$$T_r = \frac{Jd\omega}{dt} \qquad (9)$$

$$T_v = K_f\omega \qquad (10)$$

where K_m is the armature or torque constant, J is the inertia of rotor and pump impeller masses, K_f is the damping coefficient associated with the rotating members of the motor-pump system and ω is the angular velocity;

The shaft torque required to rotate the pump impeller is given as [15];

$$T_p = \rho Q(r_2V_{\theta 2} - r_1V_{\theta 1}) \qquad (11)$$

where ρ is the water density, Q is the pump flow rate, r_1, r_2 are the inner and outer radius of the impeller, $V_{\theta 2}$and $V_{\theta 2}$ are the tangential components of the absolute velocity, U . If $B = \rho(r_2V_{\theta 2} - r_1V_{\theta 1})$ then, $T_p = BQ$, hence;

$$\frac{d\omega}{dt} = \frac{K_m}{J}i - \frac{K_f}{J}\omega - \frac{BQ}{J} \qquad (12)$$

For two or more pumps of the same design having geometric similarity, the effective flow rate, Q is given as [15];

$$Q = \omega\frac{r_m{}^3}{8} \qquad (13)$$

Where $r_m = \frac{r_1+r_2}{2}$ is the mean radius of the pump impeller. The Laplace transforms of these equations of the system with initial conditions at zero, are given as follows;

$$V_a(s) = RI(s) + sLI(s) + K_b\omega(s) \qquad (14)$$

$$s\omega(s) = \frac{K_m}{J}I(s) - \frac{K_f}{J}\omega(s) - \frac{B}{J}Q(s) \qquad (15)$$

$$Q(s) = \omega(s)\frac{r_m{}^3}{8} \qquad (16)$$

Then, solving for $I(s)$ and $\omega(s)$ gives;

$$I(s) = \frac{V_a(s) - K_b\omega(s)}{(sL+R)} \qquad (17)$$

$$\omega(s) = \frac{8Q(s)}{r_m{}^3} \qquad (18)$$

Eventually, the transfer function for the pump-motor relationship that yield voltage as input to the system and pump flow rate as the output is given as;

$$\frac{Q(s)}{V_a(s)} = \frac{K_m r_m{}^3}{8LJs^2 + (8LK_f + BLr_m{}^3 + 8JR)s + (8RK_f + BRr_m{}^3 + 8JK_b)} \qquad (19)$$

B. Modeling of the Irrigation Sprinkler Nozzle

The irrigation sprinkler nozzle serves as the water distribution medium for the real-time automatic irrigation system. The sprinkler head design sketch is given in Figure 2.

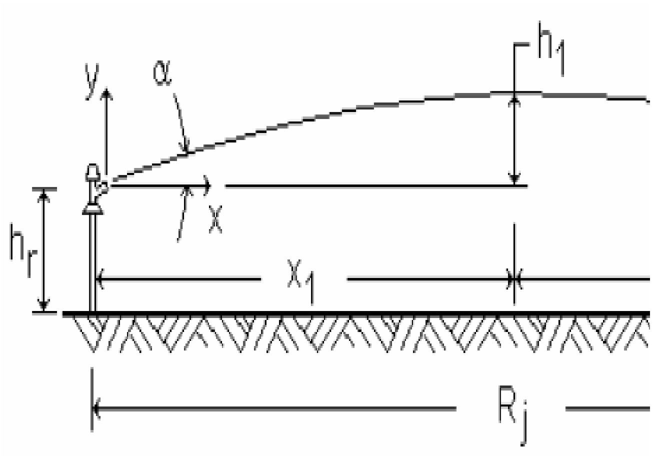

Figure 2: The sprinkler head design sketch

The fixed head sprinkler irrigation system is adopted for the analysis. Let the nozzle orifice discharge flow rate be denoted as Q_s , the sprinkler pressure be denoted as P_s and the sprinkler nozzle area is denoted as A_r, then the flow rate is given as ;

$$Q_s = V_j A_r \quad (20)$$

Then, based on Bernoulli's equation, from Figure 2, the Sprinkler head, H is given as;

$$H = z_s = \frac{P_s}{\rho_w g} = \frac{V_j{}^2}{2g} = \frac{Q_s{}^2}{2A_r{}^2 g} \quad (21)$$

Where z_s is the sprinkler elevation, ρ_w is water density, and V_j is the water discharge velocity.

If the elevations are the same $(z_{s1} = z_{s2})$ and the conversion through the nozzle is assumed to be all pressure to all velocity, then

$$Q_s = \sqrt{\frac{2A_r{}^2 P_s}{\rho_w}} = K_d\sqrt{P_s} = K_o A_r \sqrt{P_s} \quad (22)$$

Where $k_d = \sqrt{\frac{2A_r{}^2}{\rho_w}}$ and $K_o = \sqrt{\frac{2}{\rho_w}}$, K_o is the orifice coefficient and A_r is the nozzle bore area. The value of K_o is fairly consistent across nozzle sizes for a specific model and manufacturer. The nozzle diameter (bore) is given as;

$$d_s = \sqrt{\frac{4Q_s}{\pi K_o \sqrt{P_s}}} \quad (23)$$

C. Sprinkler Application Rate, Precipitation Rate and Gross Depth

The gross application depth is the total equivalent depth of water which must be delivered to the field to replace all or part of the soil moisture deficit in the root zone of the plant, plus any seepage, evaporation, spray drift, runoff and deep percolation losses. The gross application rate d_g is given as;

$$d_g = \begin{cases} \frac{d_n}{E_{pa}}, & for \quad LR \leq 0.1 \\ \frac{0.9 d_n}{(1-LR)E_{pa}}, & for \ LR \geq 0.1 \end{cases} \quad (24)$$

Where d_n is the net irrigation rate, E_{pa} is application efficiency and LR is the leaching rate. The net irrigation depth is best determined locally by checking how much water is given per irrigation application with the local irrigation method and practice. The net irrigation depth is dependent on the root depth of the crop and on the soil type.

D. Modeling the Irrigation System Capacity

Application volume can be expressed as either Qt or $A_a d_g$, where Q is flow rate, t is time, A_a is irrigated area and d_g is gross application depth. Both terms are in units of volume and thus, the system capacity, Q_T is defined as

$$Q_T = K_t \frac{A_a d_g}{fT} = n_s Q_s \quad (25)$$

Where T is hours of system operation per day (where, $T \leq 24$, and $t = Tf$), K_t is coefficient for conversion of units, n_s is the number of sprinkler heads and f is the time to complete one irrigation cycle (days). The value of K_t in metric unit for d_g in mm, A_a in ha, and Q_T in lpm, $K_t = 2.78$. Also, in imperial units and for d_g in inches, A_a in acres, and Q_T in gpm, $K_t = 453$.

Moreover, the average application rate , I_r is calculated as;

$$I_r = \frac{3600 Q_s R_e}{S_e S_i} \quad (26)$$

where I_r is the application rate (mm/hr); Q_s is the flow rate (lps); S_e is the sprinkler spacing (m); S_i is the lateral spacing (m); and R_e is the fraction of water emitted by the nozzle that reaches the soil, taking into account the evaporative and wind losses. The sprinkler precipitation rate, P_r is given as;

$$P_r = \frac{I_r}{E_{pa}} \text{ or } E_{pa} = \frac{I_r}{P_r} \quad (27)$$

If the emphasis is laid on leaching requirement where $LR \leq 0.1$ then, from Equations (27) gross depth, d_g in Equation (24) becomes;

$$d_g = \frac{d_n P_r}{I_r} \quad (28)$$

Substituting I_r from Equation (26) into Equation (28) gives;

$$d_g = \frac{d_n P_r S_e S_i}{3600 Q_s R_e} \quad (29)$$

Now, from Equation (25), $Q_s = \frac{Q_T}{n_s}$, then substituting for Q_s in Equation (29) gives;

$$d_g = \frac{d_n P_r S_e S_i n_s}{3600 Q_T R_e} \quad (30)$$

Also, from Equation (22) and Equation (25) Q_T is given as;

$$Q_T = K_t \frac{A_a d_g}{fT} = n_s Q_s = K_o A_r n_s \sqrt{P_s} \quad (31)$$

The solving for the gross depth of sprinkler in Equation (31) gives;

$$d_g = \frac{K_o A_r f T n_s \sqrt{P_s}}{K_t A_a} \quad (32)$$

The solving for the precipitation rate, P_r from Equation (30) and Equation (32) gives;

$$P_r = \frac{3600 K_o A_r f T R_e \sqrt{P_s} Q_T}{K_t d_n} \quad (33)$$

By defining the precipitation rate as the rate of change of the gross depth with respect to time, then Equation (33) becomes;

$$\frac{d(d_g)}{dt} = \frac{3600 K_o A_r f T R_e \sqrt{P_s} Q_T}{K_t d_n} \quad (34)$$

The transfer function for the sprinkler gross depth in Equation (34) with initial conditions at zero and the flow rate considered as the input is given as:

$$\frac{d_g(s)}{Q_T(s)} = \frac{3600 K_o A_r f T R_e \sqrt{P_s}}{(K_t d_n)s} \quad (35)$$

E. Modeling of the plant water uptake process

In normal circumstances, the amount of water within a plant is set by a balance between the water uptake through the roots and the rate of transpiration through the leaves. If the water saturation is below a level termed the 'permanent wilting point', the water uptake process ceases, and the plant wilts. However, if irrigation is very high, the plant roots can become waterlogged. If water logging persists for long periods of time, the roots can start rotting and the plant will eventually die. Clearly, the rate of water uptake by plants and the factors which control this are of fundamental interest from an agricultural point of view. Water goes from the roots to the leaves and shoots along the xylem tubes located in the central part of the root. Xylem vessels are tube-like structures formed of non-living cells which provide mechanical support to the roots and stems, and transport the water and nutrient ions.

Water flow along the xylem tubes is characterized by Poiseuille law for water flow in a cylindrical tube. Therefore, Q_z which isthe axial flux of water (downwards) inside the root is given as the sum of all the fluxes in each open and functional xylem tube [16]:

$$Q_z = -k_z \left[\frac{p_r}{z} - \rho_g g\right] \quad (36)$$

where p_r is the root fluid pressure inside the xylem tubes, z is the position along the root, and the quantityk_z is expressed as [17]:

$$k_z = \sum_{i=0}^{n} \frac{\pi n_i R_i^4}{8\mu} \quad (37)$$

where k_z is known as axial conductivity in soil science, R_i is the radius of the xylem vessel, $n_i$is the number of open functional xylem vessels with radius R_i per cross section of the root, and i is an index for different radius categories. The unit of Q_z is m^3/s; and that of k_z is m^4/Pa/s, and μ is dynamic viscosity of fluid. If the water fluxes along the root length is redefined as $Q_z = \frac{\partial\theta}{\partial t}$, then ;

$$\frac{\partial\theta}{\partial t} = -k_z \left[\frac{p_r}{z} - \rho_g g\right] \quad (38)$$

Where θ is the soil moisture into the root of the plant. However, if the actual water uptake by plant is P_w, then:

$$\frac{\partial\theta}{\partial t} = -k_z \left[\frac{p_r}{z} - \rho_g g\right] - P_w \quad (39)$$

Now,

$$\frac{\partial\theta}{\partial t} = T(t) = e^{qt} \quad (40)$$

Where;

$$q = \lambda = \frac{-c^2}{4D} + \left(\frac{\pi}{L}\right)^2 \quad (41)$$

Then,

$$e^{qt} = e^{\frac{-c^2 t}{4D} + \left(\frac{\pi}{L}\right)^2 t} = -k_z \left[\frac{p_r}{z} - \rho_g g\right] - P_w \quad (42)$$

The soil permeability k_z is different from hydraulic conductivity K by scaling factor which is given by,

$$K = \frac{k_z \rho_g g}{\mu} \quad (43)$$

Then,

$$k_z = \frac{K(\mu)}{\rho_g g} \quad (44)$$

The solving for the plant water uptake , P_w on Equation (42) gives;

$$P_w = -\frac{K\mu}{\rho_g g}\left[\frac{p_r}{z} - \rho_g g\right] - e^{\frac{-c^2 t}{4D} + \left(\frac{\pi}{L}\right)^2 t} \quad (45)$$

III Conclusion

The analytical models for various subsystems of a DC motor driven pump sprinkler irrigation nozzle and plant water uptake process for application in automatic irrigation process is studied. Particularly, the mathematical models for the DC motor driven irrigation water pump were derived along with the models for sprinkler irrigation nozzle, the models for the sprinkler application rate, precipitation rate and gross depth and the models for the plant water uptake process. The models presented in this paper are relevant for the simulation and parametric analysis of the DC motor driven pump sprinkler irrigation nozzle and plant water uptake process for automatic irrigation process.

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