

Gravity and Acceleration Produced in a Curved Space

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Abstract— Gravity is generated by curvature of space, as is well known in General Relativity. However, there is no clear mechanism of why gravity as force is generated and acceleration is generated when the space curves. Given a priori assumption that space as a vacuum has a physical fine structure like continuum, it enables us to apply a continuum mechanics to the so-called “vacuum” of space. The pressure field derived from the geometrical structure of space is newly obtained by applying both continuum mechanics and General Relativity to space. This paper is an attempt to explain the cause of gravity and acceleration produced in a curved space. Furthermore, this paper is a systematic compilation of the contents of the author's papers and books so far on gravity and acceleration generated in curved space regions [1-6].

Keywords— Curvature; gravity; General Relativity; space-time; continuum mechanics; acceleration field; curved space.

1. INTRODUCTION

The phenomenon of falling objects has attracted people since the days of ancient Greece.

Universal gravity, or gravity, makes everyone interested in the essence of the phenomenon.

Why do apples fall? Why is there an attractive force between two objects? Fortunately, the formula of universal gravitational force was derived by Isaac Newton in 1665, and is phenomenologically used to explain observed facts and widely used in astronomical mechanics and spacecraft orbit calculations. However, it is still unknown why and how gravity is generated.

The author tried to explain the cause of gravity by applying the mechanical structure of space to General Relativity, and by applying continuum mechanics to space. The apples on the Earth will not be pulled by the Earth and fall, but will be pushed and fall in the direction of the Earth due to the pressure of the field in the curved space area around the Earth.

Given a priori assumption that space as a vacuum has a physical fine structure like continuum, it enables us to apply a continuum mechanics to the so-called “vacuum” of space. Minami proposed a hypothesis for

mechanical property of space-time in 1988 [1]. A primary motive was to research in the realm of space propulsion theory using the substantial physical structure of space-time based on this hypothesis.

When we make a comparison between the space on the Earth and outer space, although there seems to be no difference, obviously a different phenomenon occurs. Simply put, an object moves radially inward, that is, drops straight down on the Earth, but in the outer space, the object floats and does not move.

The difference between the two phenomena can be explained by whether space is curved or not, that is, whether 20 independent components of a Riemann curvature tensor is zero or not. In essence, the existence of spatial curvature and curved extent region determine whether the object drops straight down or not. Although the spatial curvature at the surface of the Earth is very small value, i.e., $1.71 \times 10^{-23} (1/m^2)$, it is enough value to produce 1G (9.8 m/s^2) acceleration.

Newton's law of universal gravitation is usually stated as that every objects attracts every other object in the universe with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers.

As is well known, the equation for universal gravitation

thus takes the form:
$$F = G \frac{m_1 m_2}{r^2};$$

where F is the gravitational force acting between two objects, m_1 and m_2 are the masses of the objects, r is the distance between the centers of their masses, and G is the gravitational constant. This gravitational force is indirectly derived from Kepler's law based on the state of one object m_1 rotating around the other object m_2 .

Originally this attraction force F (i.e., gravitational force) between the two objects must be directly derived while the two objects (m_1 and m_2) are stationary.

We introduce that the nature of the curved spatial region itself creates gravity and gravitational acceleration that push an object as field pressure.

This paper is a systematic compilation of the contents of the author's papers and books so far on gravity and acceleration generated in curved space regions [1-6].

The following Chapter 2 explains the **Gravity Produced in Curved Space**, Chapter 3 introduces **Acceleration Produced in Curved Space**, and Chapter 4 presents a vision for **Application to Outer Space**.

2. Gravity Produced in Curved Space

2.1 Generation of Surface Force Induced by Curved Space

On the supposition that space is an infinite continuum, continuum mechanics can be applied to the so-called "vacuum" of space. This means that space can be considered as a kind of transparent field with elastic properties.

Consider a thin layer of a single space obtained by slicing the space of a transparent rubber block.

If space curves, then an inward normal stress " $-P$ " is generated. This normal stress, i.e. surface force serves as a sort of pressure field as shown in Fig. 2.1.

$$-P = N \cdot (2R^{00})^{1/2} = N \cdot (1/R_1 + 1/R_2) \quad (2.1)$$

where N is the line stress, R_1, R_2 are the radius of principal curvature of curved surface, and R^{00} is the major component of spatial curvature.

A large number of curved thin layers form the unidirectional surface force as shown in Fig. 2.2.

When surface forces are accumulated, a surface force field, that is, a force field is created. An object in the force field is accelerated by the force, so an acceleration field is generated. A large number of curved thin layers form the unidirectional surface force, i.e. acceleration field. Accordingly, the spatial curvature R^{00} produces the acceleration field α .

It is now understood that the membrane force on the curved surface and each principal curvature generates the normal stress " $-P$ " with its direction normal to the curved surface as a surface force. The normal stress " $-P$ " acts towards the inside of the surface as shown in Fig. 2.1.

A thin-layer of curved surface will take into consideration within a spherical space having a radius of R and the principal radii of curvature that are equal to the radius ($R_1=R_2=R$). Since the membrane force N (serving as the line stress) can be assumed to have a constant value, Eq.(2.1) indicates that the curvature R^{00} generates the inward normal stress P of the curved surface. The inwardly directed normal stress serves as a pressure field.

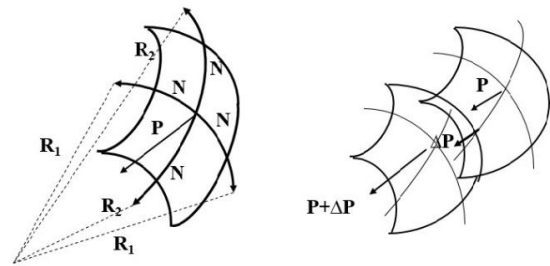


Fig. 2.1. Curvature of space plays a significant role. If space curves, then inward stress (surface force) " P " is generated \Rightarrow A sort of pressure field.

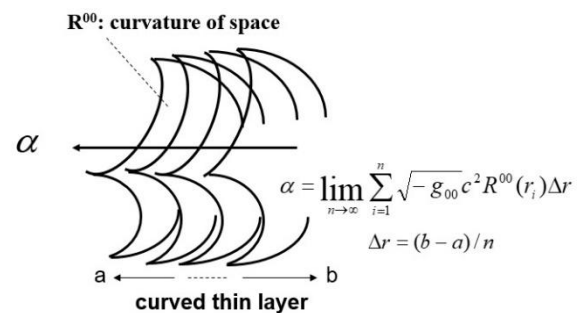


Fig. 2.2. A large number of curved thin layers form the unidirectional surface force, i.e. acceleration field α .

When the curved surfaces are included in a great number, some type of unidirectional pressure field is formed. A region of curved space is made of a large number of curved surfaces and they form the field as a unidirectional surface force (i.e. normal stress). Since the field of the surface force is the field of a kind of force, the force accelerates matter in the field, i.e., we can regard the field of the surface force as the acceleration field. A large number of curved thin layers form the unidirectional acceleration field (Fig. 2.2).

Accordingly, the spatial curvature R^{00} produces the acceleration field α .

For example, consider a soap bubble.

The pressure " P " due to the membrane force on the surface of a soap bubble of radius R is directed inward. The membrane force on the surface of the soap bubble corresponds to N in the Fig. 2.1.

$$-P = N \cdot (1/R_1 + 1/R_2) = N \cdot (1/R + 1/R) = 2N/R \quad (2.2)$$

This pressure " P " keeps the soap bubbles from breaking due to the expansion force of the internal air.

Next considering the dynamics of the surface of a soap bubble, we can see the similarity of gravity generation.

Fig. 2.1 shows the basic concept of the gravity generation mechanism. We show that the curvature of space creates a pressure field and an acceleration field.

As explained before, Fig. 2.1 shows that the vertical force P (surface force) toward the center of the surface is generated by the membrane force (line stress) N and the radii of curvature R_1 and R_2 of the thin layer with a curved space. The radius of curvature decreases toward the inner side, and the vertical stress P (surface force) increases. The surface force of the membrane layer becomes the membrane pressure.

Fig. 2.3 shows the surface force toward the center of the soap bubble.

The surface of the soap bubble expands due to surface tension (line stress N) to maintain the shape of the soap bubble, but it is known from continuum mechanics that the force toward the center of the soap bubble is actually working. If this is applied to the space as it is, the space curved in a spherical shape applies pressure toward the center of the sphere. This is the reality of gravity.



Fig. 2.3. Surface force toward the center of the soap bubble.

Fig. 2.2 shows that when a large number of curved thin film layers are integrated, unidirectional surface forces are integrated to form an acceleration field. It can be used to calculate the gravitational acceleration. Fig. 2.4 shows how the surface force due to the accumulation of many curved surfaces pushes the apple.

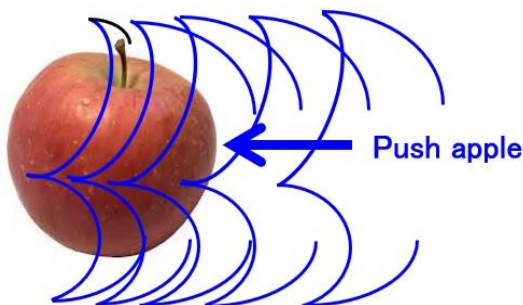


Fig. 2.4. Surface forces of multiple curved thin film layers push apple.

The fundamental three-dimensional space structure is determined by quadratic surface structure. Therefore, Gaussian curvature K in two-dimensional Riemann space is significant. The relationship between Gaussian curvature K and the major component of spatial curvature R^{00} is given by:

$$K = \frac{R_{1212}}{(g_{11}g_{22} - g_{12}^2)} = \frac{1}{2} \cdot R^{00}, \quad (2.3)$$

where R_{1212} is non-zero component of Riemann curvature tensor.

Applying membrane theory, the following equilibrium conditions are obtained in quadratic surface, given by:

$$N^{\alpha\beta} b_{\alpha\beta} + P = 0, \quad (2.4)$$

where $N^{\alpha\beta}$ is a membrane force, i.e. line stress of curved space, $b_{\alpha\beta}$ is second fundamental metric of curved surface, and P is the normal stress on curved surface [10].

The second fundamental metric of curved space $b_{\alpha\beta}$ and principal curvature $K_{(i)}$ has the following relationship using the metric tensor $g_{\alpha\beta}$,

$$b_{\alpha\beta} = K_{(i)} g_{\alpha\beta}. \quad (2.5)$$

Therefore we get:

$$N^{\alpha\beta} b_{\alpha\beta} = N^{\alpha\beta} K_{(i)} g_{\alpha\beta} = g_{\alpha\beta} N^{\alpha\beta} K_{(i)} = N_{\alpha}^{\alpha} K_{(i)} = N \cdot K_{(i)}. \quad (2.6)$$

From Eq.(2.4) and Eq.(2.6), we get:

$$N_{\alpha}^{\alpha} K_{(i)} = -P. \quad (2.7)$$

As for the quadratic surface, the indices α and i take two different values, i.e. 1 and 2, therefore Eq.(2.7) becomes:

$$N_1^1 K_{(1)} + N_2^2 K_{(2)} = -P. \quad (2.8)$$

where $K_{(1)}$ and $K_{(2)}$ are principal curvature of curved surface and are inverse number of radius of principal curvature (i.e. $1/R_1$ and $1/R_2$).

The Gaussian curvature K is represented as:

$$K = K_{(1)} \cdot K_{(2)} = (1/R_1) \cdot (1/R_2). \quad (2.9)$$

Accordingly, suppose $N_1^1 = N_2^2 = N$, we get:

$$N \cdot (1/R_1 + 1/R_2) = -P. \quad (2.10)$$

It is now understood that the membrane force on the curved surface and each principal curvature generate the normal stress “ $-P$ ” with its direction normal to the curved surface as a surface force. The normal stress $-P$ is towards the inside of surface as showing in Fig. 2.1.

A thin-layer of curved surface will be taken into consideration within a spherical space having a radius of R and the principal radii of curvature which are equal to the radius ($R_1=R_2=R$). From Eqs. (2.3) and (2.9), we then get:

$$K = \frac{1}{R_1} \cdot \frac{1}{R_2} = \frac{1}{R^2} = \frac{R^{00}}{2} \quad (2.11)$$

Considering $N \cdot (2/R) = -P$ of Eq.(2.10), and substituting Eq.(2.11) into Eq.(2.10), the following equation is obtained:

$$-P = N \cdot \sqrt{2R^{00}} \quad (2.12)$$

Since the membrane force N (serving as the line stress) can be assumed to have a constant value, Eq.(2.12) indicates that the curvature R^{00} generates the inward normal stress P of the curved surface. The inwardly directed normal stress serves as a kind of pressure field. When the curved surfaces are included in great number, some type of unidirectional pressure field is formed. A region of curved space is made of a large number of curved surfaces and they form the field of unidirectional surface force (i.e. normal stress). Since the field of surface force is the field of a kind of force, a body in the field is accelerated by the force, i.e. we can regard the field of surface force as the acceleration field. Accordingly, the cumulated curved region of curvature R^{00} produces the acceleration field α .

Here, we give an account of curvature R^{00} in advance. The solution of metric tensor $g^{\mu\nu}$ is found by gravitational field equation as the following:

$$R^{\mu\nu} - \frac{1}{2} \cdot g^{\mu\nu} R = -\frac{8\pi G}{c^4} \cdot T^{\mu\nu} \quad (2.13)$$

where $R^{\mu\nu}$ is the Ricci tensor, R is the scalar curvature, G is the gravitational constant, c is the speed of light, $T^{\mu\nu}$ is the energy momentum tensor. Furthermore, we have the following relation for scalar curvature R :

$$R = R^\alpha_\alpha = g^{\alpha\beta} R_{\alpha\beta}, R^{\mu\nu} = g^{\mu\alpha} g^{\nu\beta} R_{\alpha\beta}, R_{\alpha\beta} = R^j_{\alpha j\beta} = g^{ij} R_{i\alpha j\beta} \quad (2.14)$$

Ricci tensor $R^{\mu\nu}$ is represented by:

$$R_{\mu\nu} = \Gamma^\alpha_{\mu\alpha,\nu} - \Gamma^\alpha_{\mu\nu,\alpha} - \Gamma^\alpha_{\mu\nu} \Gamma^\beta_{\alpha\beta} + \Gamma^\alpha_{\mu\beta} \Gamma^\beta_{\nu\alpha} \quad (= R_{\nu\mu}) \quad (2.15)$$

where Γ^i_{jk} is Riemannian connection coefficient.

If the curvature of space is very small, the term of higher order than the second can be neglected, and Ricci tensor becomes:

$$R_{\mu\nu} = \Gamma^\alpha_{\mu\alpha,\nu} - \Gamma^\alpha_{\mu\nu,\alpha} \quad (2.16)$$

The major curvature of Ricci tensor ($\mu = \nu = 0$) is calculated as follows:

$$R^{00} = g^{00} g^{00} R_{00} = -1 \times -1 \times R_{00} = R_{00} \quad (2.17)$$

As previously mentioned, Riemannian geometry is a geometry that deals with a curved Riemann space, therefore Riemann curvature tensor is the principal quantity. All components of Riemann curvature tensor are zero for flat space and non-zero for curved space. If an only non-zero component of Riemann curvature tensor exists, the space is not flat space but curved

space. Therefore, the curvature of space plays a significant role.

<Supplemental explanation for Eq.(2.3)>

For a two-dimensional surface, from the Bianchi identity, the Riemann curvature tensor is given by

$$R_{\nu\mu\lambda\kappa} = K(g_{\nu\lambda} g_{\mu\kappa} - g_{\nu\kappa} g_{\mu\lambda}),$$

$$\text{that is, } R_{1212} = K(g_{11}g_{22} - g_{12}^2).$$

And, for a spherical surface of radius r , its Gaussian curvature K is $1/r^2$.

The scalar curvature R and the Gaussian curvature K on the quadratic surface are as follows:

$$\begin{aligned} R = R^i_i &= g^{ij} R_{ij} = g^{11} R_{11} + g^{12} R_{12} + g^{21} R_{21} + g^{22} R_{22} \\ &= \frac{1}{r^2}(-1) + \frac{1}{r^2 \sin^2 \theta}(-\sin^2 \theta) = -\frac{2}{r^2} = -2K \end{aligned}$$

Calculating metric and Riemannian connection coefficient in spherical coordinate system, and using

$R_{\mu\nu} = R^\sigma_{\mu\sigma\nu} = g^{\rho\sigma} R_{\rho\mu\sigma\nu}$, calculate the Ricci tensor, the following are obtained:

$$g^{11} = \frac{1}{r^2}, g^{22} = \frac{1}{r^2 \sin^2 \theta}, \text{ other } g^{\mu\nu} = 0.$$

$$R_{1212} = -r^2 \sin^2 \theta, R_{11} = -1, R_{22} = -\sin^2 \theta, R_{12} = R_{21} = 0.$$

$$\text{Namely, from } R_{\mu\nu} = g^{\rho\sigma} R_{\rho\mu\sigma\nu},$$

$$R_{11} = g^{22} R_{2121} = g^{22} R_{1212} = \frac{-r^2 \sin^2 \theta}{r^2 \sin^2 \theta} = -1,$$

$$R_{12} = g^{11} R_{1112} = 0, R_{21} = g^{11} R_{1211} = 0,$$

$$R_{22} = g^{11} R_{1212} = \frac{1}{r^2}(-r^2 \sin^2 \theta) = -\sin^2 \theta.$$

On the while, the scalar curvature R ($1/m^2$) on a four-dimensional surface is given by

$$\begin{aligned} R = R^i_i &= g_{ij} R^{ij} = g_{00} R^{00} + g_{11} R^{11} + g_{22} R^{22} + g_{33} R^{33} \\ &\approx g_{00} R^{00} = -R^{00} (g_{00} \approx -1: \text{weak field}) \end{aligned}$$

$$\text{Thus, } R = -2K = -R^{00}, \text{ then } K = \frac{1}{2} \cdot R^{00}$$

is obtained.

2.2 Mechanism of GRAVITY: as a Pressure Field Induced by Curved Space

As shown in Fig. 2.5, the gravitational field around the Earth is multiply covered by concentric or spherical curved spaces centered on the Earth.

Considering the case of the Earth, the curvature of space is spherically symmetric about the Earth and is fixed to the Earth, so the Earth itself cannot move due to the curvature of the space generated by the Earth.

However, as shown in Fig. 2.6, the apples on the Earth are independently in the curved spatial region of the Earth. Since the apple exists in the curved spatial region from the curved spatial layer at the apple's position to the curved spatial layer at the distant position, the apple is pushed by the generated curved space (i.e., pressure) and falls. That is, referring to Fig. 2.7, a sort of graduated pressure field is generated by the curved range from an arbitrary point "a" in curved space to a point "b" (the point at which space is absent of curvature, i.e., flat space of curvature 0). Then apple moves directly towards the center of the Earth, that is, the apple falls. Falling acceleration of apple in curved space is proportional to both the value of spatial curvature and the size of curved space.

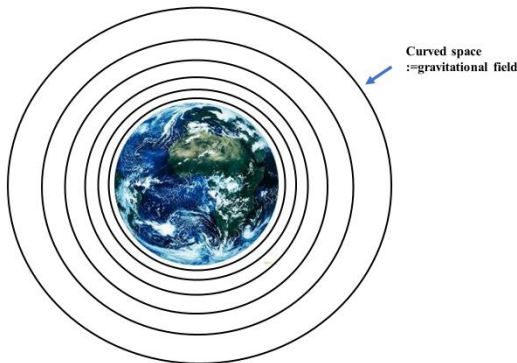


Fig. 2.5. Gravitational field around the Earth is a curved space that is concentric or spherical about the Earth.

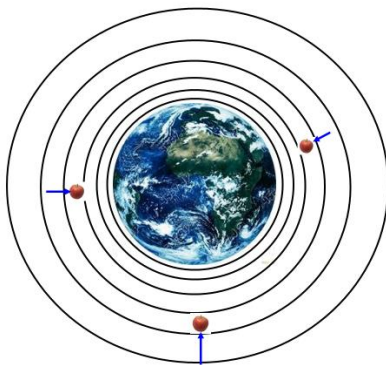


Fig. 2.6. Since the apples on the Earth are independently in the curved spatial regions of the Earth, the apples fall under the pressure generated in the curved spatial regions.

Next, consider the universal gravitational force of a well-known apple falling to the Earth. Although the attraction between the Earth and the apple by universal gravitation can be explained by a mathematical formula, $F = G \frac{Mm}{r^2}$, there is no explanation of the mechanism of the attraction, that is, the principle of operation. The mechanism can be understood by interpreting that the Earth and the apple are pushed toward each

other from behind the curved space area around the Earth and the curved space area around the apple. A phenomenon is that an apple is not pulled and falls by the Earth, but the apple is pushed toward the Earth under the pressure of the vast curved space area of the Earth.

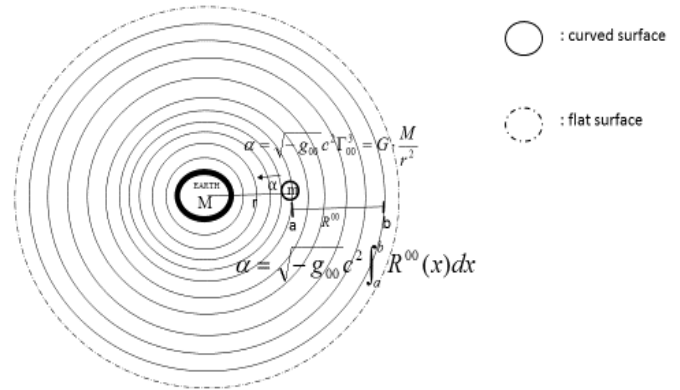


Fig. 2.7. Apple falls receiving a pressure of the field.

Fig. 2.8 shows the mechanism. In the upper diagram of Fig. 2.8, there are mass bodies A and B, and the space around each mass body is curved. As already explained, the mass B is pushed out of the curved space field generated by the mass A, and the mass A is pushed out of the curved space field generated by the mass B, so that they will move in the direction of opposition to each other. In the lower diagram of Fig. 2.8, mass A is a giant mass of the Earth, and mass B is a light apple. Apple is pushed from the vast curved space area of the Earth and go straight to the Earth. On the other hand, the Earth is also pushed from the narrow curved space area of the apple and go straight to the apple. Since the mass of an apple is smaller than that of the Earth, the range of the curved space is small and the acceleration with respect to the Earth is almost zero. In effect, it looks like an apple is pulled by the Earth and falls. Please refer to Ref. [5] in detail.

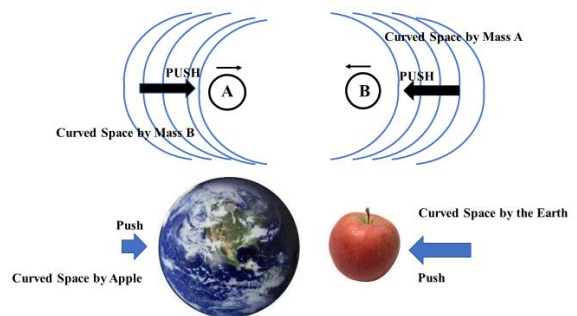


Fig. 2.8. Apple and the Earth are pushed out of a curved space and collide.

3. Acceleration Produced in Curved Space

3.1 Derivation of Acceleration

A massive body causes the curvature of space-time around it, and a free particle responds by moving along a geodesic line in that space-time. The path of free particle is a geodesic line in space-time and is given by the following geodesic equation;

$$\frac{d^2 x^i}{d\tau^2} + \Gamma_{jk}^i \cdot \frac{dx^j}{d\tau} \cdot \frac{dx^k}{d\tau} = 0 \quad (3.1)$$

where Γ_{jk}^i is Riemannian connection coefficient, τ is proper time, x^i is four-dimensional Riemann space, that is, three dimensional space ($x=x^1, y=x^2, z=x^3$) and one dimensional time ($w=ct=x^0$), where c is the velocity of light. These four coordinate axes are denoted as x^i ($i=0, 1, 2, 3$).

Proper time is the time to be measured in a clock resting for a coordinate system. We have the following relation derived from an invariant line element ds^2 between Special Relativity (flat space) and General Relativity (curved space):

$$d\tau = \sqrt{-g_{00}} dx^0 = \sqrt{-g_{00}} c dt \quad (3.2)$$

From Eq.(3.1), the acceleration of free particle is obtained by

$$\alpha^i = \frac{d^2 x^i}{d\tau^2} = -\Gamma_{jk}^i \cdot \frac{dx^j}{d\tau} \cdot \frac{dx^k}{d\tau} \quad (3.3)$$

As is well known in General Relativity, in the curved space region, the massive body " m (kg)" existing in the acceleration field is subjected to the following force F^i (N) :

$$F^i = m \Gamma_{jk}^i \cdot \frac{dx^j}{d\tau} \cdot \frac{dx^k}{d\tau} = m \sqrt{-g_{00}} c^2 \Gamma_{jk}^i u^j u^k = m \alpha^i \quad (3.4)$$

where u^j, u^k are the four velocity, Γ_{jk}^i is the Riemannian connection coefficient, and τ is the proper time.

From Eqs.(3.3),(3.4), we obtain:

$$\alpha^i = \frac{d^2 x^i}{d\tau^2} = -\Gamma_{jk}^i \cdot \frac{dx^j}{d\tau} \cdot \frac{dx^k}{d\tau} = -\sqrt{-g_{00}} c^2 \Gamma_{jk}^i u^j u^k \quad (3.5)$$

Eq.(3.5) yields a more simple equation from the condition of linear approximation, that is, weak-field, quasi-static, and slow motion (speed $v \ll$ speed of light c : $u^0 \approx 1$):

$$\alpha^i = -\sqrt{-g_{00}} \cdot c^2 \Gamma_{00}^i \quad (3.6)$$

On the other hand, the major component of spatial curvature R^{00} in the weak field is given by

$$R^{00} \approx R_{00} = R_{0\mu 0}^\mu = \partial_0 \Gamma_{0\mu}^\mu - \partial_\mu \Gamma_{00}^\mu + \Gamma_{0\mu}^\nu \Gamma_{\nu 0}^\mu - \Gamma_{00}^\nu \Gamma_{\nu\mu}^\mu \quad (3.7)$$

In the nearly Cartesian coordinate system, the value of $\Gamma_{\nu\rho}^\mu$ are small, so we can neglect the last two terms in Eq.(3.7), and using the quasi-static condition we get

$$R^{00} = -\partial_\mu \Gamma_{00}^\mu = -\partial_i \Gamma_{00}^i \quad (3.8)$$

From Eq.(3.8), we get formally

$$\Gamma_{00}^i = -\int R^{00}(x^i) dx^i \quad (3.9)$$

Substituting Eq.(3.9) into Eq.(3.6), we obtain

$$\alpha^i = \sqrt{-g_{00}} c^2 \int R^{00}(x^i) dx^i \quad (3.10)$$

Accordingly, from the following linear approximation scheme for the gravitational field equation:(1) weak gravitational field, i.e. small curvature limit, (2) quasi-static, (3) slow-motion approximation (i.e., $v/c \ll 1$), and considering range of curved region, we get the following relation between acceleration of curved space and curvature of space:

$$\alpha^i = \sqrt{-g_{00}} c^2 \int_a^b R^{00}(x^i) dx^i \quad (3.11)$$

where α^i : acceleration (m/s^2), g_{00} : time component of metric tensor, a-b: range of curved space (m), x^i : components of coordinate ($i=0,1,2,3$), c : velocity of light, R^{00} : major component of spatial curvature($1/m^2$). Eq.(3.11) indicates that the acceleration field α^i is produced in curved space. The intensity of acceleration produced in curved space is proportional to the product of spatial curvature R^{00} and the length of curved region.

Eq.(3.4) yields more simple and effective equation from above-stated linear approximation ($u^0 \approx 1$),

$$F^i = m \sqrt{-g_{00}} c^2 \Gamma_{00}^i u^0 u^0 = m \sqrt{-g_{00}} c^2 \Gamma_{00}^i = m \alpha^i = m \sqrt{-g_{00}} c^2 \int_a^b R^{00}(x^i) dx^i \quad (3.12)$$

Setting $i=3$ (i.e., direction of radius of curvature: r), we get Newton's second law:

$$F^3 = F = m \alpha = m \sqrt{-g_{00}} c^2 \int_a^b R^{00}(r) dr = m \sqrt{-g_{00}} c^2 \Gamma_{00}^3 \quad (3.13)$$

The acceleration (α) of curved space and its Riemannian connection coefficient (Γ_{00}^3) are given by:

$$\alpha = \sqrt{-g_{00}} c^2 \Gamma_{00}^3, \quad \Gamma_{00}^3 = \frac{-g_{00,3}}{2g_{33}} \quad (3.14)$$

where c : velocity of light, g_{00} and g_{33} : component of metric tensor, $g_{00,3} : \partial g_{00} / \partial x^3 = \partial g_{00} / \partial r$. We choose the spherical coordinates " $ct=x^0, r=x^3, \theta=x^1, \varphi=x^2$ " in space-time. The acceleration α is represented by the equation both in the differential form and in the integral form. Practically, since the metric is usually given by the solution of gravitational field equation, the differential form has been found to be advantageous.

<Supplemental explanation for Eq.(3.2, 3.5)>

For the flat space (Special Relativity), invariant line element ds^2 is described in

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu .$$

In the case of dx^μ is time-like,

$$ds^2 = \eta_{00} dx^0 dx^0 = -(dx^0)^2 = -(d\tau)^2 .$$

Here, $\eta_{00} = -1$ $dx^0 = d\tau = cdt$.

$d\tau$ is a proper time measured by a clock that is stationary with respect to the coordinate system. The proper time does not depend on the coordinate system.

For the curved space (General Relativity), invariant line element ds^2 is described in

$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu .$$

In the case of dx^μ is time-like,

$$ds^2 = g_{00} dx^0 dx^0 = g_{00} (dx^0)^2 .$$

We have the following relation derived from an invariant line element ds^2 between Special Relativity (flat space) and General Relativity (curved space):

since the infinitesimal line element ds^2 is invariant,

$$-(d\tau)^2 = g_{00} (dx^0)^2 .$$

then using one dimensional time ($w=ct=x^0$), we get:

$$d\tau = \sqrt{-g_{00}} dx^0 = \sqrt{-g_{00}} dw = \sqrt{-g_{00}} cdt .$$

By performing the second derivate,

$$d^2\tau = \sqrt{-g_{00}} (dx^0)^2 = \sqrt{-g_{00}} dw^2 = \sqrt{-g_{00}} c^2 dt^2 .$$

Further, as is known well in Special Relativity, in the case of slow-motion approximation (i.e., $v/c \ll 1$),

four velocity $u^\mu = \frac{dx^\mu}{d\tau}$ becomes the following:

$$u^i = \frac{1}{c} \gamma v^i \approx 0, \quad u^0 = \gamma = \sqrt{1 - (v/c)^2} \approx 1 \quad (v \ll c) .$$

Here, substitute the above equation into Eq. (3.3),

$$\alpha^i = \frac{d^2 x^i}{d\tau^2} = -\Gamma^i_{jk} \cdot \frac{dx^j}{d\tau} \cdot \frac{dx^k}{d\tau} = -\Gamma^i_{jk} u^j u^k .$$

Namely,
$$\frac{d^2 x^i}{d\tau^2} = \frac{d^2 x^i}{\sqrt{-g_{00}} c^2 dt^2} = -\Gamma^i_{jk} u^j u^k ,$$

then we get,
$$\frac{d^2 x^i}{dt^2} = -\sqrt{-g_{00}} c^2 \Gamma^i_{jk} u^j u^k .$$

Eq.(3.4):
$$F^i = m\alpha^i = m \frac{d^2 x^i}{d\tau^2} = m\Gamma^i_{jk} \cdot \frac{dx^j}{d\tau} \cdot \frac{dx^k}{d\tau} .$$

$$= m\sqrt{-g_{00}} c^2 \Gamma^i_{jk} u^j u^k$$

In the non-relativistic Newton approximation,

$$\alpha^i = \frac{d^2 x^i}{dt^2} = -\sqrt{-g_{00}} c^2 \Gamma^i_{jk} u^j u^k .$$

Eq.(3.5) yields a more simple equation from the condition of linear approximation, that is, weak-field,

quasi-static, and slow motion (speed $v \ll$ speed of light c : $u^0 \approx 1$):

$$\alpha^i = -\sqrt{-g_{00}} \cdot c^2 \Gamma^i_{00} .$$

3.2 Derivation of the Formula of Universal Gravitation

Now in general, the line element is described in:

$$ds^2 = g_{ij} dx^i dx^j = g_{00} (dx^0)^2 + g_{33} (dx^3)^2 + g_{11} (dx^1)^2 + g_{22} (dx^2)^2$$

$$= g_{00} (cdt)^2 + g_{33} (dr)^2 + g_{11} r^2 (d\theta)^2 + g_{22} r^2 \sin^2 \theta (d\varphi)^2$$

(3.15)

We choose the spherical coordinates " $ct=x^0, r=x^3, \theta=x^1, \varphi=x^2$ " in space-time (see Fig. 3.1).

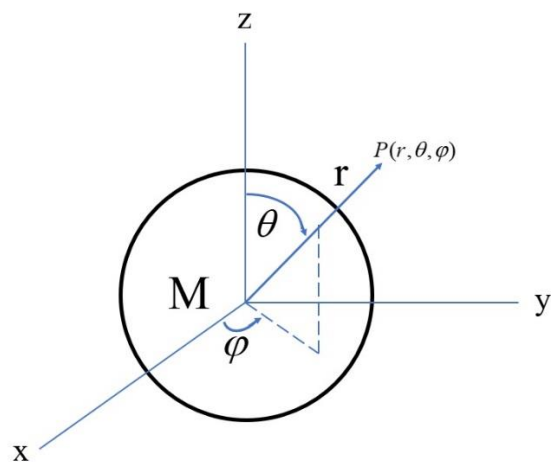


Fig. 3.1. Spherical coordinate system.

Next, let us consider External Schwarzschild Solution.

External Schwarzschild Solution is an exact solution of the gravitational field equation, which describes the gravitational field outside the spherically symmetric, static mass distribution.

The line element is obtained as follows:

$$ds^2 = -(1 - \frac{r_g}{r}) c^2 dt^2 + \frac{1}{1 - \frac{r_g}{r}} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

(3.16)

The metrics are given by:

$$g_{00} = -(1 - r_g/r), g_{11} = g_{22} = 1, g_{33} = 1/(1 - r_g/r), \text{ and other } g_{ij} = 0 .$$

(3.17)

where r_g is the gravitational radius (i.e.

$$r_g = 2GM / c^2) .$$

Combining Eq.(3.17) with Eq.(3.14) yields:

$$\alpha = G \cdot \frac{M}{r^2}, (r_g \ll r) ,$$

(3.18)

where G is gravitational constant and M is total mass.

Eq.(3.18) indicates the acceleration at a distance “r” from the center of the well-known the Earth mass M. The force acting on a mass “m” located at a distance “r” from the center of the Earth mass M is:

$$F = m\alpha = mG \frac{M}{r^2} = G \frac{Mm}{r^2} \quad (3.19)$$

Eq.(3.19) indicates a universal gravitational force acting on masses M and m that are stationary with respect to each other.

<Supplemental explanation for Eq.(3.18)>

Eq.(3.14): $\alpha = \sqrt{-g_{00}}c^2\Gamma_{00}^3$, $\Gamma_{00}^3 = \frac{-g_{00,3}}{2g_{33}}$

The metrics are given by (Eq.3.17)

$$g_{00} = -\left(1 - \frac{r_g}{r}\right) = -1 + \frac{r_g}{r}, \quad g_{33} = \frac{1}{1 - \frac{r_g}{r}}$$

Using Eq.(3.14),

$$g_{00,3} = \frac{\partial g_{00}}{\partial x^3} = \frac{\partial g_{00}}{\partial r} = \frac{\partial}{\partial r} \left(-1 + \frac{r_g}{r}\right) = -\frac{r_g}{r^2}$$

$$\Gamma_{00}^3 = \frac{-g_{00,3}}{2g_{33}} = \frac{r_g}{r^2} \cdot \frac{1 - \frac{r_g}{r}}{2} = \frac{r_g}{2r^2} - \frac{r_g^2}{2r^3} \approx \frac{r_g}{2r^2}$$

Since $r_g = \frac{2GM}{c^2}$ is the gravitational radius, then

$r_g \ll r$. Accordingly, the term of $\frac{r_g^2}{2r^3}$ is neglected.

Also, $g_{00} = -\left(1 - \frac{r_g}{r}\right) \approx -1$.

Acceleration (α) is obtained as Eq.(3.18) :

$$\alpha = \sqrt{-g_{00}}c^2\Gamma_{00}^3 = c^2\Gamma_{00}^3 = c^2 \frac{r_g}{2r^2} = c^2 \frac{1}{2r^2} \frac{2GM}{c^2} = \frac{GM}{r^2}$$

Table.1 Equations effective for the calculation of General Relativity

$\Gamma_{mmm} = (\Gamma_{mmm}) = -\frac{1}{2}g_{m,m}$	$\Gamma_{mn}^m = \Gamma_{nm}^m = \frac{g_{m,m,n}}{2g_{mm}}$
$\Gamma_{mmm} = \frac{1}{2}g_{m,m}$	$\Gamma_{mm}^m = \frac{g_{m,m,m}}{2g_{mm}}$
$\Gamma_{mnn} = (\Gamma_{mnn}) = \frac{1}{2}g_{n,n}$	$\Gamma_{nn}^m = -\frac{g_{n,n,m}}{2g_{mm}}$
$\Gamma_{mnn} = (\Gamma_{mnn}) = \frac{1}{2}g_{m,n}$	$other \quad \Gamma_{\nu l}^{\mu} = 0 \quad (\because g_{\mu\nu} = 0 \quad \mu \neq \nu)$
$other \quad \Gamma_{\nu l}^{\mu} = 0$	
$R_{\mu\nu\lambda} = \frac{1}{2}(g_{\mu l, \nu k} - g_{\nu l, \mu k} - g_{\mu k, \nu l} + g_{\nu k, \mu l}) + \Gamma_{\beta\mu\lambda}\Gamma_{\nu k}^{\beta} - \Gamma_{\beta\mu k}\Gamma_{\nu l}^{\beta}$	

By the way, from $\Gamma_{nn}^m = -\frac{g_{n,n,m}}{2g_{mm}}$,

we used $\Gamma_{00}^3 = -\frac{g_{00,3}}{2g_{33}}$ (see Table.1).

3.3 Gravitational Acceleration on the Earth's Surface

3.3.1 Overview of the Linear Approximation of Weak Static Gravitational Fields

The acceleration α and major curvature R^{00} are given by

$$R^{00} = \frac{1}{2}g^{ij}h_{00,ij}, \quad \alpha = c^2\Gamma_{00}^i = \frac{1}{2}c^2h_{00,i} \quad (3.20)$$

respectively from the weak field approximation of the gravitational field equation.

Here, h_{00} is deviation between metric tensor g_{00} of curved space and Minkowski metric tensor η_{00} of flat space, that is,

$$g_{00} = \eta_{00} + h_{00} = -1 + h_{00} \quad (3.21)$$

The notation of the symbol is as follows:

$$h_{00,ij} = \partial_i\partial_j h_{00} = \frac{\partial h_{00}}{\partial x^i\partial x^j} \quad (3.22)$$

As is well known, the partial derivative $u_{i,j} = \partial_j u_i = \frac{\partial u_i}{\partial x^j}$ is not tensor equation. The covariant derivative $u_{i;j} = u_{i,j} - u_k\Gamma_{ij}^k$ is tensor equation and can be carried over into all coordinate systems.

If the gravitational field is time-invariant, or static, and the gravitational field is not very strong, Ricci tensor $R_{\mu\nu}$ is given by:

$$R_{\mu\nu} = \frac{1}{2}(\square h_{\mu\nu} + h_{,\mu\nu} - h_{\mu\rho,\rho\nu} - h_{\nu,\rho\mu}^{\rho})$$

$$= \frac{1}{2}(\square h_{\mu\nu} + \partial_{\mu}\partial_{\nu}h - \partial_{\rho}\partial_{\nu}h_{\mu\rho} - \partial_{\mu}\partial_{\rho}h_{\nu}^{\rho})$$

where $\square = \eta^{\mu\nu}\partial_{\mu}\partial_{\nu} = \nabla^2 - (\partial_0)^2$.

$$(3.23)$$

Since all are static $h_{\mu\nu,0} = 0$ ($\partial_0 h_{\mu\nu} = 0$) and now setting $\mu = \nu = 0$, this component R_{00} is obtained

$$R_{00} = \frac{1}{2}(\nabla^2 h_{00} - (h_{00,0})^2 + h_{00} - h_{0\rho,\rho 0} - h_{0,0\rho}^{\rho}) = \frac{1}{2}\nabla^2 h_{00} \quad (3.24)$$

On the other hand,

$$g_{00} = \eta_{00} + h_{00} = -1 + h_{00} = -1 - \frac{2}{c^2}\phi \quad (3.25)$$

As is well known, potential ϕ is

$$\phi = -\frac{GM}{R} \quad (3.26)$$

where M is the Earth mass, R is the Earth radius, G is the Gravity constant.

We get,

$$h_{00} = -\frac{2}{c^2}\phi = -\frac{2}{c^2} \times -\frac{GM}{R} = \frac{2GM}{c^2 R} \quad (3.27)$$

Then,

$$R_{00} = \frac{1}{2}\nabla^2 h_{00} = \frac{1}{2}\nabla^2 \left(-\frac{2\phi}{c^2}\right) = -\frac{1}{c^2}\nabla^2 \phi \quad (3.28)$$

Curvature R_{00} can be described by the following approximation:

$$R_{00} = \frac{1}{2}\nabla^2 h_{00} = \frac{1}{2}\left(\frac{\partial^2}{\partial x^2} h_{00} + \frac{\partial^2}{\partial y^2} h_{00} + \frac{\partial^2}{\partial z^2} h_{00}\right) \quad (3.29)$$

$$\approx \frac{1}{2} \frac{d^2 h_{00}}{dx^2} \approx \frac{1}{2} h_{00} / R^2$$

where x is toward the Earth center.

A similar result is obtained from Eq.(3.20) as:

$$\begin{aligned} R^{00} &= 1/2 \cdot g^{ij} h_{00,ij} = 1/2 \cdot g^{33} \partial^2 h_{00} / \partial x^3 \partial x^3 = 1/2 \cdot g^{33} \partial^2 h_{00} / \partial x \partial x \\ &= 1/2 \cdot \partial^2 h_{00} / \partial r^2 \approx 1/2 \cdot h_{00} / R^2 \end{aligned} \quad (3.30)$$

The approximate expression for gravitational acceleration is:

$$\alpha = \frac{1}{2} c^2 h_{00,i} = \frac{1}{2} c^2 h_{00,x} = \frac{1}{2} c^2 \frac{dh_{00}}{dx} \approx \frac{1}{2} c^2 h_{00} / R \quad (3.31)$$

On the other hand, as described in previously (see Eq.3.11), the gravitational acceleration is also given by the following equation:

$$\alpha = \sqrt{-g_{00}} c^2 \int_a^b R^{00}(r) dr \quad (3.32)$$

Considering $g_{00} = -1$, substituting Eq.(3.29) or Eq.(3.30) into Eq.(3.32), we get

$$\alpha = c^2 \int_R^\infty \frac{1}{2} \frac{h_{00}}{r^2} dr = \frac{c^2}{2} \frac{h_{00}}{R} \quad (3.33)$$

Eq.(3.33) matches Eq.(3.31), and the equation of gravitational acceleration expressed by Eq.(3.32) gives the mechanism of gravity. This physical concept becomes clear in the next section.

Further, major curvature of Ricci tensor ($\mu = \nu = 0$) is calculated as follows:

$$R^{00} = g^{00} g^{00} R_{00} = -1 \times -1 \times R_{00} = R_{00} \quad (3.34)$$

Here for convenience, raise the index and use it in the notation of R^{00} instead of R_{00} .

$$\begin{aligned} R^{00} &= \frac{1}{2} g^{ij} h_{00,ij} = \frac{1}{c^2} g^{ij} \left(\frac{1}{2} c^2 h_{00,i} \right)_{,j} \\ &= \frac{1}{c^2} g^{ij} \alpha_{i,j} = \frac{1}{c^2} \alpha^j_{,j} \end{aligned} \quad (3.35)$$

$$\text{where } h_{00,ij} = \frac{\partial h_{00}}{\partial x^i \partial x^j}$$

3.3.2 Gravitational Acceleration on the Earth

The accumulation of surface forces in a curved area of space from the Earth's surface R to the point at infinity (∞) gives the gravitational acceleration on the Earth's surface.

$$\begin{aligned} \alpha &= c^2 \int_R^\infty R^{00}(r) dr = c^2 \int_R^\infty \frac{1}{2} h_{00} \frac{1}{r^2} dr = -\frac{1}{2} c^2 h_{00} \left[\frac{1}{r} \right]_R^\infty \\ &= -\frac{1}{2} c^2 h_{00} (0 - \frac{1}{R}) = \frac{1}{2} \frac{c^2 h_{00}}{R} \end{aligned} \quad (3.36)$$

From Eq. (3.27), the deviation h_{00} of the metric tensor g_{00} from the flat space ($\eta_{00} = -1$) on the Earth's surface becomes:

$$h_{00} = \frac{2GM}{c^2 R} \quad (3.37)$$

The curvature of the space is (see Eq.3.29):

$$R_{00} = \left(\frac{1}{2} h_{00} \right) / R^2 \quad (3.38)$$

The gravitational acceleration is (see Eq.3.31):

$$\alpha = \left(\frac{1}{2} c^2 h_{00} \right) / R \quad (3.39)$$

Substitute the values of the Earth radius $R=6.378 \times 10^3 \text{ km}$, $GM=3.986 \times 10^5 \text{ km}^3/\text{s}^2$, $c=3 \times 10^5 \text{ km}$, we get the following values respectively:

$$\begin{aligned} h_{00} &= \frac{2GM}{c^2 R} = \frac{2 \times 3.986 \times 10^5}{(3 \times 10^5)^2 \times 6.378 \times 10^3} = \frac{2 \times 3.986 \times 10^5}{9 \times 6.378 \times 10^{13}} = 1.389 \times 10^{-9} \\ R_{00} &= \left(\frac{1}{2} h_{00} \right) \times \frac{1}{R^2} = \frac{1}{2} \times 1.389 \times 10^{-9} \times \frac{1}{(6.378 \times 10^3)^2} = 1.71 \times 10^{-2} \times 10^{-9} \times 10^{-6} \\ &= 1.71 \times 10^{-17} (1/\text{km})^2 = 1.71 \times 10^{-23} (1/m^2) \\ \alpha &= \left(\frac{1}{2} c^2 h_{00} \right) \times \frac{1}{R} = \frac{1}{2} \times (3 \times 10^5)^2 \times 1.389 \times 10^{-9} \times \frac{1}{6.378 \times 10^3} \\ &= \frac{1}{2} \times 9 \times 1.389 \times \frac{1}{6.378} \times 10^{10} \times 10^{-9} \times 10^{-3} = 0.98 \times 10^{-2} \text{ km} / \text{s}^2 = 9.8 \text{ m} / \text{s}^2 \end{aligned}$$

In this way, the following approximate values can be obtained:

The amount of displacement of the space on the Earth's surface: $h_{00} = 1.389 \times 10^{-9}$.

Curvature of space on the Earth's surface:

$$R_{00} = 1.71 \times 10^{-23} / m^2$$

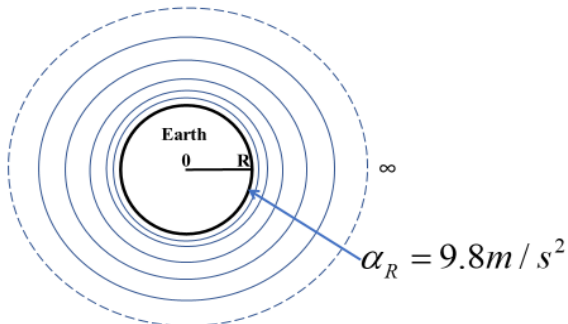
Gravitational acceleration on the Earth's surface

$$\alpha = 9.8m / s^2.$$

Next, a description will be given using the drawings. Fig. 3.2 shows a concentric curved space area around the Earth. Distance of radius R from the center of the Earth is the surface of the Earth. At a distance from the Earth to an infinite point, the space becomes flat space without being affected by the gravitation of the Earth. The point at infinity is indicated by a symbol ∞ and a dotted line. The accumulation of surface forces in a curved area of space from the Earth's surface R to the point at infinity (∞) gives gravitational acceleration on the Earth's surface, i.e., $\alpha_R = 9.8m / s^2$.

$$\alpha = c^2 \int_R^\infty R^{00}(r) dr = c^2 \int_R^\infty \frac{1}{2} h_{00} \frac{1}{r^2} dr = -\frac{1}{2} c^2 h_{00} \left[\frac{1}{r} \right]_R^\infty$$

$$= -\frac{1}{2} c^2 h_{00} \left(0 - \frac{1}{R} \right) = \frac{1}{2} \frac{c^2 h_{00}}{R} \tag{3.40}$$



$$\alpha = c^2 \int_R^\infty R^{00}(r) dr = c^2 \int_R^\infty \frac{1}{2} h_{00} \frac{1}{r^2} dr = -\frac{1}{2} c^2 h_{00} \left[\frac{1}{r} \right]_R^\infty = -\frac{1}{2} c^2 h_{00} \left(0 - \frac{1}{R} \right) = \frac{1}{2} \frac{c^2 h_{00}}{R}$$

Fig. 3.2. Mechanism of gravitational acceleration generation on the Earth's surface.

As well, Fig. 3.3 shows a concentric curved space area around the Earth. Distance of radius R from the center of the Earth is the surface of the Earth. Here consider the gravitational acceleration at a height h away from the Earth's surface. At a distance from the Earth to an infinite point, the space becomes flat space without being affected by the gravitation of the Earth. The point at infinity is indicated by a symbol ∞ and a dotted line. The accumulation of surface forces in a curved area of space from the Earth's surface R+h to the point at infinity (∞) gives the gravitational acceleration at the Earth's height h, i.e., $\alpha_{R+h} < \alpha_R = 9.8m / s^2$.

$$\alpha = c^2 \int_{R+h}^\infty R^{00}(r) dr = c^2 \int_{R+h}^\infty \frac{1}{2} h_{00} \frac{1}{r^2} dr = -\frac{1}{2} c^2 h_{00} \left[\frac{1}{r} \right]_{R+h}^\infty$$

$$= -\frac{1}{2} c^2 h_{00} \left(0 - \frac{1}{R+h} \right) = \frac{1}{2} \frac{c^2 h_{00}}{R+h} \tag{3.41}$$

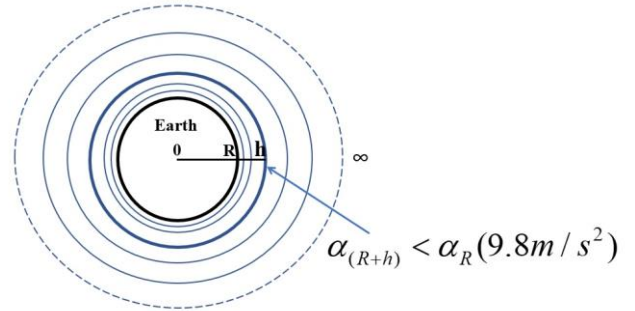


Fig. 3.3. Mechanism of gravitational acceleration generation on the Earth's surface height h.

As described above, although the spatial curvature at the surface of the Earth is very small value, i.e., $1.71 \times 10^{-23} (1/m^2)$, it is enough value to produce 1G ($9.8 m/s^2$) acceleration.

4. Application to Outer Space

Comparing the space on the ground and the space in outer space, although there seems to be no difference, obviously a different phenomenon occurs. Simply put, an object moves radially inward, that is, drops straight down on the Earth, but in the outer space, the object floats and does not move.

The difference between the two phenomena can be explained by whether space is curved or not. In essence, the existence of spatial curvature and curved extent region determine whether the object drops straight down or not. Although the spatial curvature at the surface of the Earth is very small value, i.e., $1.71 \times 10^{-23} (1/m^2)$, it is enough value to produce 1G ($9.8 m/s^2$) acceleration.

Conversely, the spatial curvature in the outer space is zero, therefore any acceleration is not produced. Accordingly, if the spatial curvature of a localized area containing object is controlled to the curvature of $1.71 \times 10^{-23} (1/m^2)$ with a sufficiently large curved space area, the object moves and receives 1G acceleration in the outer space. Of course, we are required to control both the magnitude of the curvature and the size of the curved space area.

So how can we curve the space artificially? As a matter of fact, space curvature is generated not only by mass energy but also by electromagnetic energy. From General Relativity, the major component of curvature of space R^{00} can be produced by not only mass density but also the magnetic field B as follows:

$$R^{00} = \frac{4\pi G}{\mu_0 c^4} \cdot B^2 = 8.2 \times 10^{-38} \cdot B^2 \quad (B \text{ in Tesla}), \tag{4.1}$$

where $\mu_0 = 4\pi \times 10^{-7} (H/m)$, $\epsilon_0 = 1/(36\pi) \times 10^{-9} (F/m)$,

$G = 6.672 \times 10^{-11} (N \cdot m^2 / kg^2)$, $c = 3 \times 10^8 (m/s)$,

B is a magnetic field in Tesla and R^{00} is a major component of spatial curvature ($1/m^2$).

Eq.(4.1) indicates that the major component of spatial

curvature can be controlled by a magnetic field B. In case that the intensities of the magnetic field B and the electric field E are equal, the value of $(1/2 \cdot \epsilon_0 E^2)$ is about seventeen figures smaller than the value of $(B^2/2\mu_0)$. As a consequence, the electric field only negligibly contributes to the spatial curvature as compared with the magnetic field.

The relationship between curvature and magnetic field was derived by Minami and introduced it in 16th International Symposium on Space Technology and Science (1988) [1].

Please refer to [1, 7 (appendix A), 8 (appendix A), 9] for Eq.(4.1) derivation.

5. Conclusion

Assuming that space is an infinite continuum, a mechanical concept of space became identified. Space can be considered as a kind of transparent elastic field. The pressure field derived from the geometrical structure of space is newly obtained by applying both continuum mechanics and General Relativity to space.

The mass on the Earth will not be pulled by the Earth and fall, but will be pushed and fall in the direction of the Earth due to the pressure of the field produced in the curved space area around the Earth. Gravity and its gravitational acceleration can be explained as a pressure field induced by the curvature of space.

Acknowledgements

I sincerely thanks for their excellent books continuously guiding this research [10, 11, 12].

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