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Visualizing The Ladder Kinematic Problem

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***Abstract*—Three cases of motions are applied to a simple sliding ladder model. The mathematical relationships between the position of the bottom of ladder and the velocities of the top, and the arbitrary position of ladder can be easily obtained. The graphs with different accelerations of the bottom of the ladder sliding on the ground are shown. These graphs would be of interest to students seeking to gain a better understanding of classical kinematics problems.**

Keywords—sliding ladder; uniform motion; acceleration motion

(Some figures may appear in color only in the online journal)

I. INTRODUCTION

The problem of a ladder leaning or sliding on a wall is a standard two-dimensional exercise. It is discussed in the statics section of introductory physics textbooks [1] and journals [2-7]. Those authors focus on the static equilibrium conditions of the ladder. Kapranidis and Koo considered three variations of the sliding ladder problem, which in each case, and taken together, provide interesting insights into the ladder problem and resolve the paradox of infinite speed [4]. However, the kinematics graphs of a ladder are less shown. In this paper, we showed a visual graph of the equations of motion. Here, three cases of motion for the bottom of the ladder are discussed.

II. MODEL

The side view of the uniform ladder [7], touching a vertical wall (the *y*-axis) and a horizontal floor (the *x*-axis) meeting at the origin O, is shown in Fig. 1. For simplicity, the friction between the ladder and the wall, and the floor are ignored. The bottom and the top of ladder are defined as A and B, respectively, $AB=L$. C labels an arbitrary position on the ladder, the distance to the bottom is L_i ($AC=L_i$ $0 \le L_i \le L$).

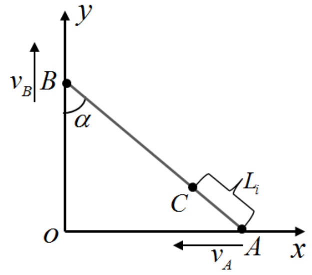

Figure 1. The geometry and coordinate system of the ladder.

s ... coordinate is ... configuration. The angle of the ladder with the wall α was chosen as the variable to describe the movement while the ladder is in contact with the wall. Then, it is assumed that the positions of A and B are (x_A ,0) and (0, y_B), respectively, at time t (or, owing to the constraints, simply x_A and y_B, respectively). It is straightforward shown that $x_A^2 + y_B^2 = L^2$. The velocity of the top B is

$$v_B = \frac{\mathrm{d}y_B}{\mathrm{d}t} = -\frac{x_A}{\sqrt{L^2 - x_A^2}} v_A \quad (1)$$

Its direction is along the positive *y*-axis. In further calculations, the components of velocity at an arbitrary position on the ladder are

$$v_{Cx} = \frac{L_i}{L} v_A , v_{Cy} = -\frac{L - L_i}{L} v_A \frac{x_A}{\sqrt{L^2 - x_A^2}} \quad (2)$$

Thus, the magnitude and the direction of the velocity of the arbitrary position C are

$$v_C = v_A\sqrt{[L(L^2+x_A^2)-2L_i x_A^2]/L(L^2-x_A^2)} \quad (3)$$

$$\tan\theta = x_A(L-L_i)/L_i\sqrt{L^2-x_A^2}, \quad (4)$$

Here, θ is the angle of the component of velocity of an arbitrary position v_{Cx} with the velocity v_C. On the other hand，the orbit equation of an arbitrary position on the ladder is obtained, $x_C^2/(L-L_i)^2+y_C^2/L_i^2=1$, which is an ellipse (the orbit reduces to a circle for the ladder's centre of mass).

A. Motion of bottom A with constant velocity

The first graph we show the bottom of the ladder is moving along the negative x direction at a constant speed. As the references [5-7] shown, when the ladder is moving under the influence of gravity alone, the bottom of the ladder does not move at constant speed along negative x direction. Therefore, in order to maintain a constant speed of the bottom along negative x direction, additional forces on the ladder are required. This motion can be realized by pushing back at the bottom. Here the initial conditions are $t=0$, $x_A = x_0$, and $v_A = v$, so the position of *A* is $x_A = x_0 - vt$ at time *t*. Based on this, the velocity of an arbitrary position on the ladder can be derived from (2)–(4).

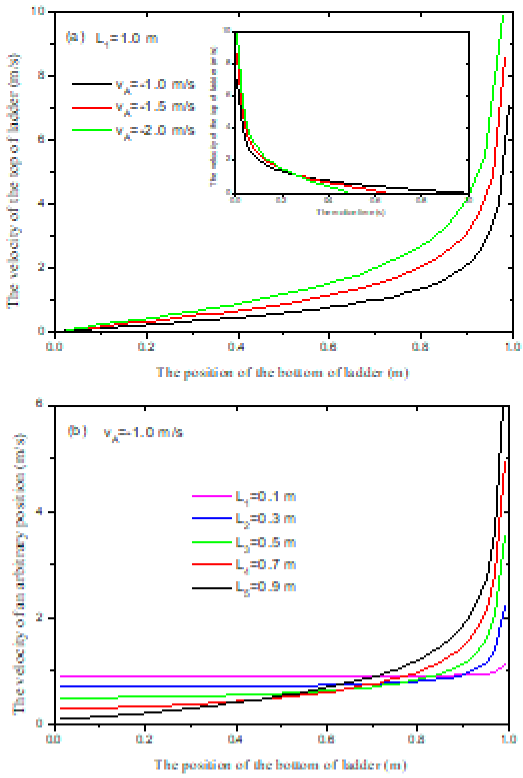

Figure 2. Panel (a) shows the velocity of ladder's top as a function of position of ladder's bottom with different velocities. Panel (b) shows the velocity of an arbitrary position as a function of position of ladder's bottom with a fixed velocity.

The velocities at top B and an arbitrary position C as a function of the position x_A of bottom *A* for different initial velocities and distances L_i are shown in Fig. 2. Here, L_i (*i*=1,2,3,4,5) represents different distances from C to A. From Fig. 2(a), it is seen that the velocity of B decreases with the position x_A decreasing, $v_B = v_A \tan\alpha$. As shown by the inset in Fig.2 (a), the whole motion time of the ladder is shortened with the increase of the speed of bottom A. At $t \le 0.32\text{s}$, the acceleration of B at $v_A = 1.0\text{m/s}$ is less than that for $v_A = 1.5\text{m/s}$ and $v_A = 2.0\text{m/s}$. In Fig. 2(b), the velocity of arbitrary position C is shown based on (3). It is seen that the velocity of position C is also decreasing when bottom A is close to the point O. However, the velocity is going to fall faster near the top B.

B. Motion of bottom A with constant acceleration

Now we turn to what happens if the acceleration of ladder's bottom is a constant. For simplicity, the initial conditions are $t=0$, $x=x_0$, and $v_{A0}=0$, respectively. The varied velocity is supposed as $v_A = Kt$ (K is a constant). Thus, the motion equation of ladder's bottom is obtained, $x_A = x_0 - Kt^2/2$, and the velocity of arbitrary position are still given by (2)–(4).

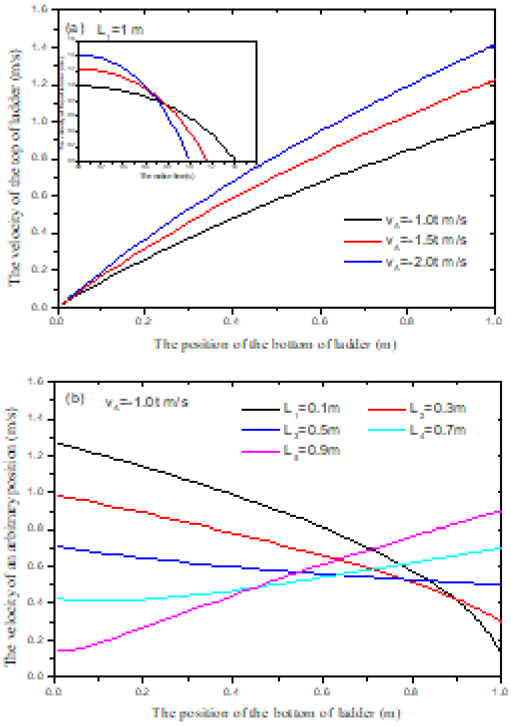

Figure 3. Panel (a) shows the velocity of ladder's top as a function of position of ladder's bottom with different constant accelerations. Panel (b) shows the velocity of an arbitrary position as a function of position of ladder's bottom with a fixed acceleration.

Figure 3 shows the dependence of the velocities of top B and an arbitrary position C on the position of bottom A. The inset in Fig. 3(a) is the velocity of top B with the motion time. In Fig. 3(a), the decreases in velocity during the first 0.2 s are slower than those in Fig. 2(a). Moreover, we can see clearly that the velocity of top B is decreasing when the bottom A is close to the point O in all regions. The dependence of the velocities at position C for different positions (0, 0.1m, 0.3m, 0.5m, 0.7m and 0.9m) on the position of bottom A is shown in Fig. 3(b). As can be seen, the variation trend of velocity at position C is different for different positions on the ladder. When the arbitrary position that we chose is near the bottom A on the ladder, the velocity of C is increasing with the bottom A closing the point O. On the contrary, this is definitely not the case of the position that close to top B, the magnitude of the velocity monotonically decreases.

C. Motion of bottom A with varied acceleration

Finally we consider the ladder where the acceleration of the bottom is not a constant. The initial conditions are the same with Fig. 3. Supposed $a_A = K't$, where K' is a constant, thus, we can obtain $x_A = x_0 - (K'/6)t^3$ and $v_A = (K'/2)t^2$. In this case, the velocities of top B and an arbitrary position C are once again determined by (1)，(2) and (4), respectively.

The dependence of the velocities of top B and an arbitrary position C under varied acceleration are shown in Fig. 4. One important difference between the constant acceleration case (Fig. 3(a)) and the varied acceleration case (Fig.4(a)) is that the curves are parabolas, especially for the inset in Fig.4(a), the whole motion time of the ladder is shortened with the increase of the velocity at bottom A. Furthermore, the maximum of the velocity B is appeared at $x_A = 0.85\mathrm{m}$. Moreover, when the arbitrary position that we chose is near the top B [Fig. 4(b)], its velocity increases firstly and then decreases.

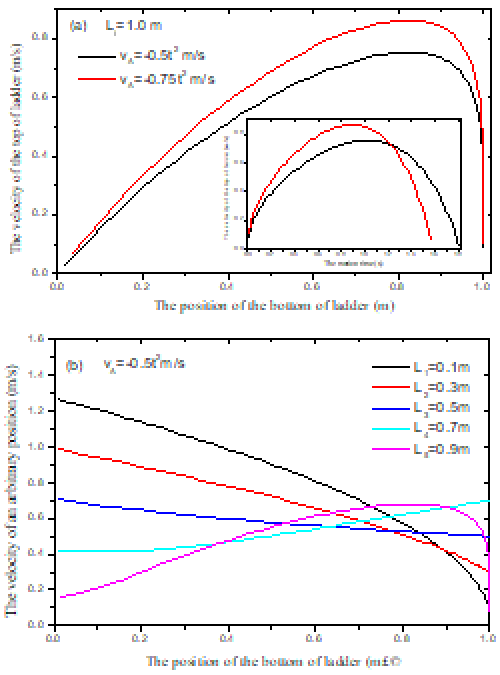

Figure 4. Panel (a) shows the velocity of ladder's top as a function of position of ladder's bottom with different changed accelerations. Panel (b) shows the velocity of an arbitrary position as a function of position of ladder's bottom with a changed acceleration.

III. CONCLUSION

In this paper, we showed a visualizing graph about the velocity of different positions on the ladder. Despite its simplicity, these graphs can help the students understand kinematic problem easier. The case in which the velocity of the bottom was constant, the velocities of the top and an arbitrary position monotonically decreased with the bottom position decreasing. However, in the cases in which the acceleration of the bottom is constant or varied, only the velocity nearby the bottom position $\left(L_i \leq 0.5\text{m}\right)$ decreased monotonically with the bottom position increasing. In a word, this is one of the simplest physics problems that exhibits a combination of theory and numerical calculation and can be useful in enhancing the understanding of classical kinematics concepts by using visualization graph.

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