Science and Technology Publishing (SCI & TECH)
ISSN: 2632-1017
Vol. 4 Issue 12, December - 2020

A Plane Positioning Formula Based On The Combined Measurement Of A Point And A Line

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***Abstract*—By geometrically compensating for path difference, a relation among the arrival angle, intersection angle, as well as path difference of the target is obtained formally, from which the geometric position of the target can be deduced.**

***Keywords*—Single point direction finding; arrival angle; path difference; intersection angle; passive location**

I. Introduction

If the continuous multiple detection mode is not considered, then no matter using single-point direction finding or single-baseline path difference measurement, generally only the target's orientation can be obtained, and the result of single-baseline path difference measurement is generally only an approximate value[1]-[7]. In this paper, a new positioning formula is given by extending the concept of geometric compensation of single baseline path difference.

The initial result of geometric compensation analysis of single baseline path difference only shows that if the field angle, that is,the intersection angle of the target corresponding to the baseline can be known, then the method utilizing the path difference measurement can be used to accurately determine the orientation of the target, but in fact the intersection angle of the target seems to be difficult to be obtained in a simple way similar to that of single point measurement only in one point.

However, the subsequent reverse analysis finds that, if the azimuth of the target is accurately measured at the reference position, and the measurement result of single-baseline path difference can also be used at the same time, the intersection angle of the target can be obtained. In fact, this result means that the plane geometric position of the target can be determined by single-point direction finding and single-baseline path difference measurement.

II. Geometric compensation for the path difference

The single-base array can only obtain one path difference, and the direction finding formula obtained based on this path difference is only an approximate solution, and can only be applied to short baselines, and cannot be extended to long baselines. The geometrical reason for the approximation is that the existing direction finding formula is calculated based on the relationship between edges and angles in a right triangle, but the path difference obtained from the actual measurement is less than the right-angle side corresponding to the arrival angle, and the path difference measurement itself is difficult to give more observable parameters.

The existing approximate direction finding formula based on single baseline is

$$\sin\theta_i \approx \frac{\Delta r_i}{d_i} \qquad (1)$$

Where: θ_i is the arrival angle of the target; Δr_i the path difference; d_i the baseline length.

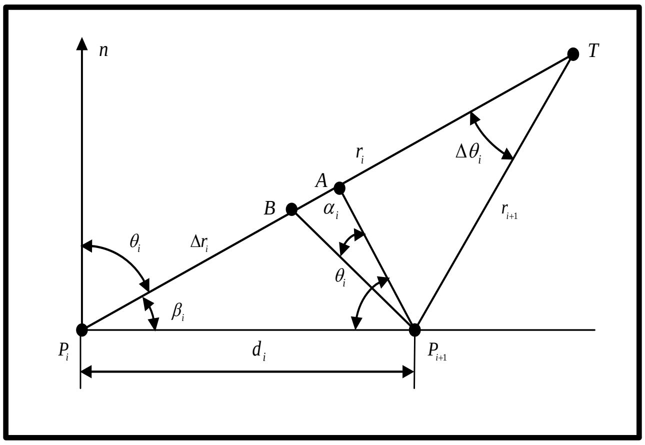

Fig. 1. *Geometric correction of the path difference.*

According to the geometric relationship shown in figure 1, in order to satisfy the relationship between side and angle of a right triangle, the length of the right angle side corresponding to the angle of arrival must be

$$P_iA = \Delta r_i + AB \qquad (2)$$

Among them, the line segment AB is used to compensate the shortage of right Angle edge. For the path difference measurement, there is a condition that the two base angles in a triangle need to be equal, which can be written as the following equation

$$0.5\left(180^0 - \Delta\theta_i\right) = \alpha_i + 90^0 - \Delta\theta_i \qquad (3)$$

Where: $\Delta\theta$ is called the intersection angle between two detection points; α compensation Angle.

Thus it can be proved that

$$\alpha_i = \angle BP_{i+1}A = 0.5\Delta\theta_i \tag{4}$$

According to the geometric relation, the compensation line segment can be solved

$$AB = d_i \cos\theta_i \cdot tg\left(0.5\Delta\theta_i\right) \tag{5}$$

The direction finding formula after compensation is as follows

$$\sin\theta_i = \frac{P_iA}{d_i} = \frac{\Delta r_i}{d_i} + \cos\theta_i \cdot tg\left(0.5\Delta\theta_i\right) \tag{6}$$

III. TARGET INTERSECTION ANGLE

Using the sine theorem, we have

$$\sin\Delta\theta_i = \frac{d}{r_{i+1}}\cos\theta_i \tag{7}$$

Using the half Angle formula, we can get

$$\mathrm{tg}\frac{\Delta\theta}{2} = \frac{\sin\Delta\theta}{1+\cos\Delta\theta} = \frac{d_i\cos\theta_i}{r_{i+1}\left(1+\sqrt{1-\left(\frac{d_i\cos\theta_i}{r_{i+1}}\right)^2}\right)} \tag{8}$$

After substituting the tangent term related to the intersection angle in equation (6), we have

$$\sin\theta_i = \frac{\Delta r_i}{d_i} + \frac{d_i\cos^2\theta_i}{r_{i+1}\left(1+\sqrt{1-\left(\frac{d_i\cos\theta_i}{r_{i+1}}\right)^2}\right)} \tag{9}$$

When the target is far away and the baseline is relatively short, The approximate have

$$\sin\theta_i = \frac{\Delta r_i}{d_i} + \frac{d\cos^2\theta_i}{2r_{i+1}} \tag{10}$$

Using the sine theorem again, through the joint solution equations (7) and (10), after eliminating the radial distance of the target, a formula for directly solving the intersection angle of the target can be obtained

$$\sin\Delta\theta_i = \frac{2}{\cos\theta_i}\left(\sin\theta_i - \frac{\Delta r_i}{d_i}\right) \tag{11}$$

Based on the expression of equation (11), it can be seen that if the precise direction finding instrument is used at the reference measurement position of a single baseline, such as the P_i point shown in figure 1, and the single baseline path difference measurement value is also used, the intersection angle of the target can be determined by the difference between the precise direction finding sine angle and the approximate direction finding sine angle. Thus, the following product difference relation can be obtained:

$$\sin\Delta\theta_i\cos\theta_i = 2\left(\sin\theta_i - \frac{\Delta r_i}{d_i}\right) \tag{12}$$

Namely, the product of intersection angle sine and target arrival angle cosine is equal to twice the difference between the precise direction finding sine angle and the approximate direction finding sine angle.

In addition, the error between the two direction finding angle values can be directly observed from the figure shown in Fig. 1. which is approximately the difference between two angle:

$$\angle P_iP_{i+1}A - \angle P_iP_{i+1}B$$

The difference is directly corresponding to the compensation angle: $\alpha_i = 0.5\Delta\theta_i$, it is also a quantity directly related to the intersection angle, which indicates that the difference between two direction finding angles of the precise and approximate is approximately half of the target intersection angle, that is:

$$\theta_i - \sin^{-1}\left(\frac{\Delta r_i}{d_i}\right) \approx 0.5\Delta\theta_i \tag{13}$$

In fact, this means that if the intersection angle is larger, the accuracy directly using the path difference direction finding is not high.

From the perspective of plane geometry analysis, target intersection angle is the difference of target azimuth measured respectively on both ends of a single baseline, and the results in this paper indicates: the intersection angle of target can also be determined by the difference between the precise direction finding angle and the approximate direction angle through the direction finding respectively with a single point and single-base line.

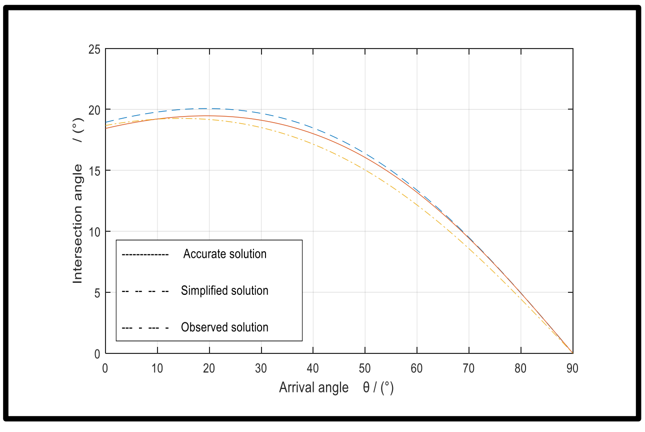

Fig. 2. *Comparison of accuracy of intersection Angle calculation.*

Fig. 2 compares the calculation accuracy of equation (11) and (13). Since equation (11) is obtained through some kind of simplified analysis, it is called the simplified solution. Formula (13) is directly observed from the figure, so it is called the observed solution. If the baseline length is small, all curves will be glued together and cannot be observed. Therefore, in order to facilitate observation, the larger value of the baseline length d=100km is deliberately adopted in the calculation.

IV. THE TARGET DISTANCE

Once the intersection angle of the target is obtained, the radial distance of the target can be obtained directly from formula (7)

$$r_{i+1} = d_i \frac{\cos\theta_i}{\sin\Delta\theta_i} \quad (14)$$

If it is necessary to carry out calculation and analysis from the reference position, then

$$r_i = d_i \frac{\sin(180^0 - \beta_i - \Delta\theta_i)}{\sin\Delta\theta_i} = d_i \frac{\sin(\beta_i + \Delta\theta_i)}{\sin\Delta\theta_i} \quad (15)$$
$$= d_i \frac{\cos(\theta_i - \Delta\theta_i)}{\sin\Delta\theta_i} = d_i(\cos\theta_i ctg\Delta\theta_i + \sin\theta_i)$$

Use the half angle formula again

$$\mathrm{ctg}\Delta\theta_i = \frac{1}{tg\Delta\theta_i} = \frac{1}{2tg\frac{\Delta\theta_i}{2}} = \frac{1+\cos\Delta\theta_i}{2\sin\Delta\theta_i}$$

Substitute the solution of the target intersection angle into the above equation

$$\mathrm{ctg}\Delta\theta_i = \frac{1+\cos\Delta\theta_i}{2\sin\Delta\theta_i} = \frac{\cos\theta_i\left[1+\sqrt{1-\frac{4}{\cos^2\theta_i}\left(\sin\theta_i - \frac{\Delta r_i}{d_i}\right)^2}\right]}{4\left(\sin\theta_i - \frac{\Delta r_i}{d_i}\right)} \quad (16)$$

Finally

$$r_i = d_i(\cos\theta_i ctg\Delta\theta_i + \sin\theta_i)$$
$$= d_i\left[\sin\theta_i + \frac{\cos^2\theta_i}{4\left(\sin\theta_i - \frac{\Delta r_i}{d_i}\right)}\left[1+\sqrt{1-\frac{4}{\cos^2\theta_i}\left(\sin\theta_i - \frac{\Delta r_i}{d_i}\right)^2}\right]\right] \quad (17)$$

V. RANGING ACCURACY

For the convenience of error analysis, the distance measurement equation (15) is directly used

$$r_i = d_i(\cos\theta_i ctg\Delta\theta_i + \sin\theta_i)$$

A. Relative ranging error generated only by measurement error of arrival angle

The derivative of the target distance with respect to the target arrival Angle obtained by single point measurement is

$$\frac{\partial r_i}{\partial\theta_i} = d_i\left(-\sin\theta_i ctg\Delta\theta_i - \cos\theta_i \csc^2\Delta\theta_i \frac{\partial\Delta\theta_i}{\partial\theta_i} + \cos\theta_i\right) \quad (18)$$

Due to

$$\sin\Delta\theta_i = \frac{2}{\cos\theta_i}\left(\sin\theta_i - \frac{\Delta r_i}{d_i}\right)$$

After differentiating both sides

$$\cos\Delta\theta_i \frac{\partial\Delta\theta_i}{\partial\theta_i} = \frac{2}{\cos^2\theta_i}\left(1 - \frac{\Delta r_i}{d_i}\sin\theta_i\right)$$

Thus

$$\frac{\partial\Delta\theta_i}{\partial\theta_i} = \frac{2}{\cos\Delta\theta_i \cos^2\theta_i}\left(1 - \frac{\Delta r_i}{d_i}\sin\theta_i\right) \quad (19)$$

The relative ranging error generated only by the measurement error of the Angle of arrival is

$$\sigma_{r\theta} = \left|\frac{\partial r}{\partial\theta}\right| \frac{\sigma_\theta}{r} \quad (20)$$

Where : σ_θ is the root mean square error of angle measurement, and the unit is radian.

B. Relative ranging error generated only by time difference measurement error

The differential of the target distance with respect to the distance difference is

$$\frac{\partial r_i}{\partial\Delta r_i} = -d_i \cos\theta_i \csc^2\Delta\theta_i \frac{\partial\Delta\theta_i}{\partial\Delta r_i} \quad (21)$$

The differential with respect to the distance difference in the two sides of the intersection angle calculation formula (11) is

$$\cos\Delta\theta_i \frac{\partial\Delta\theta_i}{\partial\Delta r_i} = -\frac{2}{d_i\cos\theta_i}$$

Can be obtained

$$\frac{\partial\Delta\theta_i}{\partial\Delta r_i} = -\frac{2}{d_i\cos\theta_i\cos\Delta\theta_i} \quad (22)$$

If it is assumed that the path difference is obtained based on the time difference measurement, then use , we have

$$\frac{\partial\Delta r}{\partial\Delta t} = v_c$$

So are

$$\frac{\partial r_i}{\partial\Delta t} = \frac{\partial r_i}{\partial\Delta r_i}\frac{\partial\Delta r_i}{\partial\Delta t} = -v_c d_i \cos\theta_i \csc^2\Delta\theta_i \frac{\partial\Delta\theta_i}{\partial\Delta r_i} \quad (23)$$

The relative ranging error generated only by the time difference measurement error is

$$\sigma_{rt} = \left|\frac{\partial r}{\partial\Delta t}\right| \frac{\sigma_t}{r} \quad (24)$$

Where: σ_t is the root mean square error of the time difference measurement.

C. Relative ranging error generated only by baseline measurement error

The derivative of the target distance with respect to the baseline length is

$$\frac{\partial r_i}{\partial d_i}=\left(\cos\theta_i ctg\Delta\theta_i+\sin\theta_i\right)-d_i\cos\theta_i\csc^2\Delta\theta_i\frac{\partial\Delta\theta_i}{\partial d_i} \quad (25)$$

By differentiating both sides of equation (11) of intersection Angle, we can get

$$\cos\Delta\theta_i\frac{\partial\Delta\theta_i}{\partial d_i}=\frac{2\Delta r_i}{d_i^2\cos\theta_i}$$

Thus, the derivative of the intersection Angle with respect to the baseline is obtained

$$\frac{\partial\Delta\theta_i}{\partial d_i}=\frac{2\Delta r_i}{d_i^2\cos\theta_i\cos\Delta\theta_i} \quad (26)$$

The relative ranging error generated only by the baseline measurement error is

$$\sigma_{rd}=\left|\frac{\partial r}{\partial d}\right|\frac{\sigma_d}{r} \quad (27)$$

Where: σ_d is the root mean square error of length measurement.

D. Relative ranging error

According to the error analysis theory, the relative ranging error obtained by dual direction finding is

$$\begin{aligned}\sigma_r&=\frac{1}{r}\left(\sigma_{r\theta}\sigma_\theta+\sigma_{rt}\sigma_t+\sigma_{rd}\sigma_d\right)\\&=\frac{1}{r}\left(\left|\frac{\partial r}{\partial\theta}\right|\sigma_\theta+\left|\frac{\partial r}{\partial\Delta t}\right|\sigma_t+\left|\frac{\partial r}{\partial d}\right|\sigma_d\right)\end{aligned} \quad (28)$$

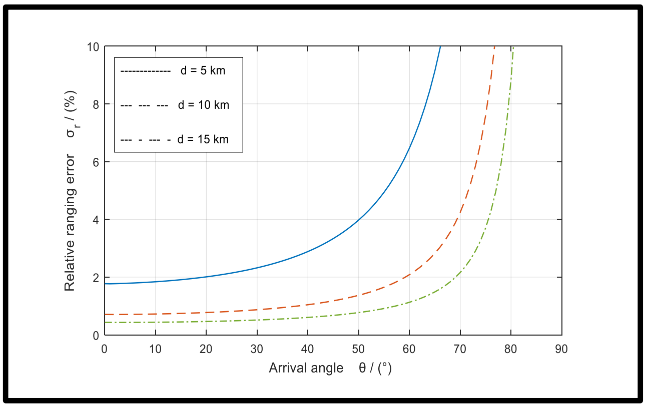

Fig. 3. *Relative ranging error.*

Fig. 3. shows the relative ranging error curve with different baseline length when the target distance is 300 km. In the simulation calculation, the root mean square of the baseline measurement error $\sigma_d=10m$, the root mean square of the time measurement error $\sigma_t=100\ ns$ and the root mean square of the azimuth measurement error are taken $\sigma_\theta=1^0\cdot\pi/180$.

Simulation analysis shows that the ranging accuracy increases with the increase of baseline length. As the Angle of arrival approaches 90 degrees, a detection blind spot will appear.

VI. CONCLUSION

By analyzing the compensation of path difference direction finding, this paper presents a positioning algorithm which combines angle and path difference measurement. According to the traditional plane geometry analysis, the geometric position of the target should be determined by observing at two different positions. The positioning method proposed in this paper also needs to adopt two different observation methods, so strictly speaking, it does not deviate from the traditional positioning mechanism. However, based on the traditional analysis method, if the combination of single point direction finding and path difference measurement is applied, it may be necessary to directly solve the higher-order equation in mathematics. Based on the concept of path difference compensation, the calculation formula can be derived directly and smoothly.

At the same time, based on the concept of path difference compensation, we have a further understanding of the relationship between the target intersection angle and the target azimuth angle at different measurement positions, leaving a lot of imagination space for people to further explore various applications.

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