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Development Of Strict Differential Seeded Secant Numerical Iteration Method For Computing The Semi Major Axis Of A Perturbed Orbit Based On The Anomalistic Period

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***Abstract*— In this paper, development of Strict Differential Seeded Secant (SDSS) numerical iteration method for computing the semi major axis of a perturbed orbit based on the anomalistic period is presented. Specifically, the value of the semi major axis (a) and the nominal mean motion (n_o) of the orbit are determined based on a given anomalistic period of the orbit. The mathematical expressions, the procedure and flowchart for the SDSS method are presented. A numerical case study shows how effective the method can be used to determine the semi major axis along with the nominal mean motion of an orbit impacted by the oblateness of the earth. The results for the case study orbit show that the nominal mean motion (no) is 1.4553996417E-04 rad/s while the mean motion (n) considering the earth oblateness is 1.4550264551E-04 rad/s which gives a difference (n-no) of 1.3579203307E-08 (rad/s). Essentially, the orbit mean motion is faster with the oblate earth and a given anomalistic period (p). The results show that the required values of the semi axis(a) and the nominal mean motion (n_o) are obtained after the first iteration of the SDSS method. Also, the orbit semi major axis at $n =$ 1.4550264551E-04 is 26598.53828 km whereas, at $no =$ 1.4548906631E-04 athe orbit semi major axis is 26604.7414 km . This gives a difference, Δa of 6.20312 km. In essence, the oblate earth cause the semi major axis of the case study orbit to increase by $= 6.20312$ km above what the value that for the perfectly spherical earth.**

Keywords*— *Oblate Earth, Orbit, Nominal Mean Motion, Anomalistic Period, Seeded Secant, Secant Method, Continuous Perturbation-Based Seeded Secant

I. INTRODUCTION

In the study of motion of satellites on Keplerian orbits where the earth is assumed to be perfectly spherical, the mean motion (in this case, referred to as nominal mean motion,n_o) is computed as a function of two parameters, namely, the earth geocentric gravitational constant (μ) and the semi major axis (a) of the orbit [1,2,3,4,5,6,7,8]. However, in reality, the earth is not a perfect sphere but an oblate spheroid with a bulge at the equator and flattening at the north and south poles [9,10,11,12,13,14]. The oblate spheroid shape of the earth affects the orbit and the motion of satellites in the orbit. In this case, the orbit is said to be perturbed [15,16,17,18,19] and the mean motion, denoted as n is a complex equation that includes several parameters among which are the orbit inclination angle (i); earth geocentric gravitational constant (μ), eccentricity (e), the nominal mean motion, (n_o) and the semi major axis (a) of the orbit. Furthermore, the period of the perturbed orbit is called anomalistic period (p) which has a simple analytical relationship with the mean motion of the perturbed orbit. In this paper, an approach for solving for the semi major axis (a) and hence the nominal mean motion, (n_o) of a perturbed orbit is presented. The solution approach computes the mean motion (n) from the knowledge of the anomalistic period (p) and then uses a modified version of secant numerical iteration method [20,21,22,23,24,25] developed in this paper to compute the semi major axis and the nominal mean motion, (n_o) of the perturbed orbit.

Specifically, the secant iteration version developed in this paper is called Strict Differential Seeded Secant (SDSS) numerical iteration method. The SDSS method requires only a single initial value from which it automatically generates a second initial value that enables it to proceed with the usual secant iteration. The second initial guess root is generated by adding a differential, (which is a small fraction of the available single root) to the single root provided for the iteration. The term 'strict' here means that this procedure of using the differential of the single root is adopted in all the cycles of the secant iteration until the algorithm converges to the desired solution. The detailed algorithm, flowchart and mathematical models associated with the new algorithm are presented along with numerical examples.

II. OVERVIEW OF STRICT DIFFERENTIAL SEEDED SECANT NUMERICAL ITERATION METHOD

Generally , the classical scant iteration requires two initial roots r(0) and r(1) (or r(k) and r(k+1)) which will be used to compute the third root estimate r(2) (or r(k+2)). In the seeded secant, only one initial guess root is required , which is r(0) (or r(k)). In the Strict Differential Seeded Secant (SDSS) numerical iteration method the second root r(k+1) is obtained simply by adding a differential of r(0) to r(0), hence r(1) = r(0) + δ(r(0) where δ is a very small fraction, much less than 1. In practice, δ is to about 0.000001 (that is 1 x10^{-6}). When r(k+1) = r(k) + δ(r(k) is used for all the k (where k is the iteration cycle number), the method is termed strict differential seeded secant. However, if r(k+1) = r(k) + δ(r(k) is used only for the initial cycle , that is , only when k = 0 , then the method is termed onetime differential seeded secant. In this second case, the classical secant method for finding root is applied for all k>0. In this paper, the strict differential seeded secant is employed in the determination of the semi major axis (a) and the nominal mean motion (n_o) of an orbit which is affected by the oblateness of the earth.

III. METHODOLOGY

Notably, in this paper, value of the semi major axis (a) and the nominal mean motion (n_o) are determined based on a given anomalistic period of the orbit. When the anomalistic period (P) is given, the orbit mean motion, denoted as n, is given as;

$$n = \frac{2\pi}{P} \tag{1}$$

Also, the nominal mean motion denoted as n_o is given in respect of the semi major axis, (a) as;

$$n_o = \sqrt{\frac{\mu}{a^3}} \tag{2}$$

In respect of n_o and the semi major axis, (a), the mean motion, (n) is given as;

$$n = n_o\left[1 + \frac{K_1(1-1.5\sin(i)^2)}{a^2(1-e^2)^{1.5}}\right] \tag{3}$$

Hence,

$$n = \left(\sqrt{\frac{\mu}{a^3}}\right)\left[1 + \frac{K_1(1-1.5\sin(i)^2)}{a^2(1-e^2)^{1.5}}\right] \tag{4}$$

$$\frac{n^2}{\left[1+\frac{K_1(1-1.5\sin(i)^2)}{a^2(1-e^2)^{1.5}}\right]^2} = \frac{\mu}{a^3} \tag{5}$$

Hence;

$$a = \left(\frac{\mu}{n^2}\left[1 + \frac{K_1(1-1.5\sin(i)^2)}{a^2(1-e^2)^{1.5}}\right]^2\right)^{1/3} \tag{6}$$

In this paper, in order to determine the nominal mean motion denoted as n_o, the semi major axis, a is determined using Strict Differential Seeded Secant (SDSS) numerical iteration method. In the SDSS method, a function of the semi major axis, denoted as f(a_k) is used to iteratively compute a for a number of cycles counted using the counter k. Let f(a_k) be defined as;

$$f(a_k) = a_k - \left(\frac{\mu}{\left(\frac{2\pi}{P}\right)^2}\left[1 + \frac{K_1(1-1.5\sin(i)^2)}{(a_k)^2(1-e^2)^{1.5}}\right]^2\right)^{1/3} = a_k - \left(\frac{\mu}{n^2}\left[1 + \frac{K_1(1-1.5\sin(i)^2)}{(a_k)^2(1-e^2)^{1.5}}\right]^2\right)^{1/3} \tag{7}$$

The seeded secant method requires only one initial value, which in this case is denoted as a_o. The initial value of a is obtained from $n_o = \sqrt{\frac{\mu}{a^3}}$, where it is assumed that the initial value of n_o = n= $\frac{2\pi}{P}$; hence $a_o = \frac{2\pi}{P}$. Once, a_o is obtained , then a fraction of the a_o is taken as a perturbation value which is added to the a_o to obtain the second value, a_1 , as required by the classical secant method. With a_o and a_1 the next value a_2 is computed by the secant method and the results is checked in respect of the set tolerance error, Є.

Accordingly, the flowchart for the Strict Differential Seeded Secant (SDSS) numerical iteration method is given in Figure 1 whereas the algorithm is stated as follows:

Step 1: The initial values of key parameters are defined.

Step 1.1: Initialize a

$$a_o = \left(\frac{\mu}{n^2}\right)^{1/3} = \left(\frac{\mu}{\left(\frac{2\pi}{P}\right)^2}\right)^{1/3} \tag{8}$$

Step 1.2: Initialize the counter, k

K = 1 (9)

Step 1.3: Initialize the perturbation parameter, δ

δ = 0.000001 (10)

Step 1.4: Initialize the error tolerance, ∈

∈ = 0.0001 (11)

Step 2: Compute the second root value a_k from the seed value a_{k-1};

$$a_k = (1 + \delta)a_{K-1} \tag{12}$$

Step 3: Compute $\mathbf{f}(\mathbf{a_{k-1}})$

$$\mathbf{f}(\mathbf{a_{k-1}}) = a_{k-1} - \left(\frac{\mu}{n^2}\left[1 + \frac{K_1(1-1.5\sin(i)^2)}{(a_{k-1})^2(1-e^2)^{1.5}}\right]^2\right)^{1/3} \tag{13}$$

Step 4: Compute $\mathbf{f}(\mathbf{a_k})$

$$\mathbf{f}(\mathbf{a_k}) = a_k - \left(\frac{\mu}{n^2}\left[1 + \frac{K_1(1-1.5\sin(i)^2)}{\mathbf{a_k}^2(1-e^2)^{1.5}}\right]^2\right)^{1/3} \tag{14}$$

Step 5: Compute $\mathbf{a_{k+1}}$

$$a_{k+1} = \frac{(a_{k-1})f(a_k) - (a_k)f(a_{k-1})}{f(a_k) - f(a_{k-1})} \tag{15}$$

Step 6: Compute $\mathbf{f}(\mathbf{a_{k+1}})$

$$\mathbf{f}(\mathbf{a_{k+1}}) = a_{k+1} - \left(\frac{\mu}{n^2}\left[1 + \frac{K_1(1-1.5\sin(i)^2)}{(\mathbf{a_{k+1}})^2(1-e^2)^{1.5}}\right]^2\right)^{1/3} \tag{16}$$

Step 7: Check if the required value of **a** has been obtained

Step 7.1: The response when the required value of **a** has been obtained

Step 7.1.1:

If $|f(a_{k+1})| \le \epsilon$ (where ϵ is error tolerance) then the actual value of a has been determined an it is a_{k+1}. At this point, $n_o$is computed using the expression

Step 7.1.2: Compute n_o

$$n_o = \sqrt{\frac{\mu}{a^3}} = \sqrt{\frac{\mu}{(a_{k+1})^3}} \tag{17}$$

Step 7.1.3: Output results

Output k , a_{k+1} , n_o

Step 7.2: The response when the required value of **a** has not been obtained

If however, $|f(a_{k+1})| > \epsilon$, then the actual value of a has not been found; in that case, the following steps are taken;

Step 7.2.1:

$$a_k = a_{k+1} \qquad (18)$$

Step 7.2.2:

$$k = k + 1 \qquad (19)$$

Step 7.2.3:

Repeat step 2 to step 7

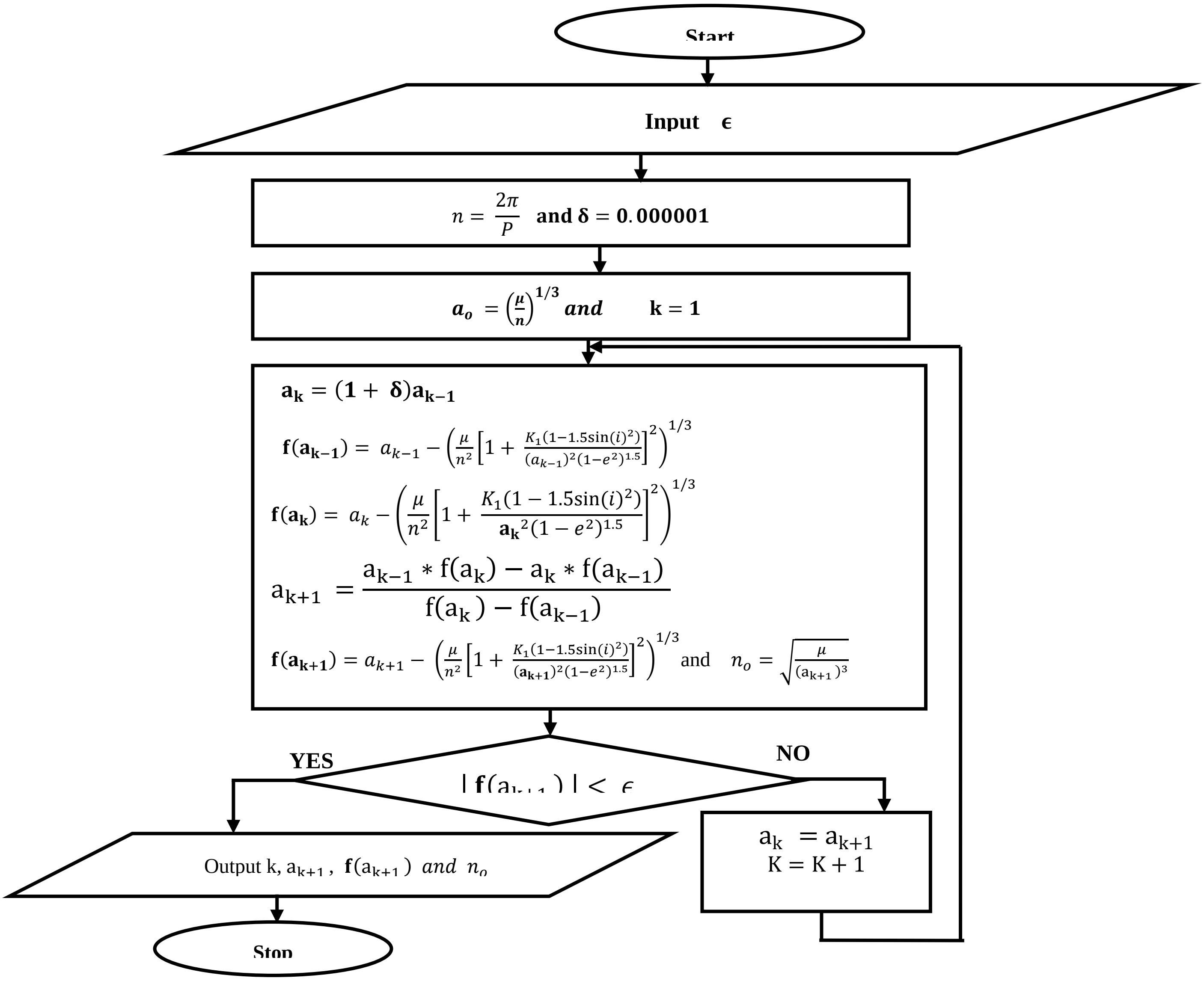

Figure 1 The flowchart for Strict Differential Seeded Secant (SDSS) numerical iteration method

IV. RESULTS AND DISCUSSION

The Strict Differential Seeded Secant (SDSS) numerical iteration method is used to determine the semi axis(a) and the nominal mean motion (n_o) of a case study orbit with the parameters given in Table 1. The result of the Strict Differential Seeded Secant (SDSS) numerical method for the computation of ak and no is shown in Table 2.The results show that the nominal mean motion (no) is 1.4553996417E-04 rad/s while the mean motion (n) considering the earth oblateness is 1.4550264551E-04 rad/s which gives a difference (n-no) of 1.3579203307E-08 (rad/s). Essentially, the orbit mean motion is faster with the oblate earth and a given anomalistic period (p). The results show that the required values of the semi axis(a) and the nominal mean motion (n_o) are obtained after the first iteration of the SDSS method.

The change in mean motion, Δn and the change in orbit semi major axis, Δa are computed from the results in Table 2 a follows;

Change in mean motion, Δn where ;

$$\Delta n = \textbf{n-no (rad/s)} = 1.3579203307\text{E-08} \textbf{ (rad/s)} \qquad (20)$$

Change in orbit semi major axis, Δa where;

At no = n = 1.4550264551E − 04;

$$a = a_n = 26598.53828 \text{ km} \qquad (21)$$

At no = 1.4548906631E − 04;

$$a = a_{no} = 26604.7414 \text{ km} \qquad (22)$$

$$\Delta a = a_{no} - a_n \qquad (23)$$

Hence;

$$\Delta a = 26604.7414 - 26598.53828 = 6.20312 \text{ km}$$

Also, the orbit semi major axis at n = 1.4550264551E − 04 = a_n = 26598.53828 km whereas, at no =

$1.4548906631E-04$; $a = a_{no} = 26604.7414$ km . This gives a difference, $\Delta a = a_{no} - a_n = 6.20312$ km. In essence, the oblate earth cause the semi major axis of the case study orbit to increase by = 6.20312 km above what the value that for the perfectly spherical earth.

Table 1 The parameters of a case study orbit

S/N	PARAMETER	SYMBOL	VALUE	UNIT
1	Eccentricity	e	0.002	
2	Earth Geocentric Gravitational Constant	μ	3.986005 x10^{14}	m^3/s^2
3	Inclination Angle	i	0	degree
4	Anomalistic Period	P	12	hour
5	Constant	K_1	66,063.1704	km^2

Table 2 The result of the Strict Differential Seeded Secant (SDSS) numerical method

	Orbit semi major axis	Estimation error in ak	Acceptable Estimation error in ak	Nominal mean motion (no)	Mean motion (n)	
Cycle	ak (km)	f(ak) (km)	ϵ	no (rad/s)	n (rad/s)	n-no (rad/s)
0	26598.53828	-6.2038947E+00	1.00E-03	1.4553996417E-04	1.4550264551E-04	-3.7318657097E-08
1	26604.7414	-2.6697671E-07	1.00E-03	1.4548906631E-04	1.4550264551E-04	1.3579201117E-08
2	26604.7414	0.0000000E+00	1.00E-03	1.4548906631E-04	1.4550264551E-04	1.3579203307E-08
3	26604.7414	0.0000000E+00	1.00E-03	1.4548906631E-04	1.4550264551E-04	1.3579203307E-08
4	26604.7414	0.0000000E+00	1.00E-03	1.4548906631E-04	1.4550264551E-04	1.3579203307E-08

V. CONCLUSION

A Strict Differential Seeded Secant (SDSS) numerical iteration method is presented for computing the value of the semi major axis along with the nominal mean motion of an orbit when the impact of the earth oblateness is considered. The mathematical expressions, the procedure and flowchart for the SDSS method are presented. As regards root finding numerical solutions, the perturbation, in this case, indicates a fraction of the current value of the root that is added to get a second root that is used to compute the next root estimate. The numerical case study shows how effective the method can be used to determine the semi major axis along with the nominal mean motion of an orbit impacted by the oblateness of the earth.

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