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Bit Error Probability Analytical Models For Multi-Level Rectangular Quadrature Amplitude Modulation Scheme

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***Abstract*— In this paper, Bit Error Probability (BER) analytical models for multi-level rectangular Quadrature Amplitude Modulation (QAM) were presented. Also, the analytical models for the Symbol Error Probability (SER) were presented. Generalized BER models for multi-level rectangular QAM modulation scheme were presented in termed of Q-function and complementary error function (erfc). Sample numerical computation of the BER for the multi-level rectangular QAM modulation scheme was performed in Matlab for modulation order ranging from 4 to 4096 and the values of the coefficients for the Q-function-based models, namely; A_{Qfn} and B_{Qfn} as well as the coefficients for the erfc-based models, namely; A_{erfc} and B_{erfc} were obtained. Notably, at modulation order, M =16, $A_{Qfn} = 3/4$ and B_{Qfn} = 4/5 , $A_{erfc} = 3/8$ and B_{erfc} =2/5. Also, the results on the BER versus Eb/No for various modulation order of the multi-level rectangular QAM showed that, at Eb/No= 4 dB: BER = 1.2501E-02 for M = 4, BER = 5.8618E-02 for M = 16, 1.1576E-01 for M = 64, BER = 1.4691E-01 for M = 256, and BER = 1.5230E-01 for M = 1024. Essentially, the BER was lowest at M= 4 and highest at M =1024. In all, BER increases with M for any given Eb/No.**

Keywords*— *Bit Error Probability, Multi-Level Rectangular QAM, Symbol Error Probability, Complementary Error Function, Modulation Order , Q-Function, Modulation Scheme, Quadrature Amplitude Modulation

1. INTRODUCTION

Quadrature Amplitude Modulation (QAM) modulation technique allows carrier signal amplitude to vary with phase [1,2,3,4,5,6,7,8]. As such, QAM is seen as a combination of Amplitude Shift Keying (ASK) and Phase Shift Keying (PSK) modulation schemes [9,10,11,12,13,14,15,16,17,18]. In comparison with other modulation schemes, QAM has higher bandwidth efficiency for a given average signal power [19,20,21,22,23,24]. In recent years, QAM modulation scheme has been among the most widely used modulation schemes in different communication systems, such as wireless local-area networks (WLANs), cable TV, wireless sensor networks (WSN), cellular telephone systems, Wi-Fi and satellites communications [25,25,26,27].

In respect of constellation diagram structure, there are different types of QAM such as rectangular constellation QAM, square constellation QAM, circular symmetry constellation QAM, etc [28,29,30,31,32,33]. The rectangular QAM is much easier to modulate and demodulate because it has regular structure that is generated by implementing amplitude modulation in phase and quadrature [34,35,36,37,38]. The rectangular constellation QAM is obtained by considering the constellation points located in a rectangular shape. Particularly, the rectangular QAM constellation is used when the transmission of odd number of bits per symbol is required. In general, the rectangular QAM constellation is sub-optimal as it does not space the constellation points maximally for any given energy. Nevertheless, the rectangular QAM is easier to modulate and demodulate when compared to the other non-rectangular QAMs. In this paper, the Bit Error Probability (BER) analytical models for multi-level rectangular QAM modulation scheme are studied. The BER models are presented in terms of Q-function, error function (erf) and complementary error function (erfc). The models are implemented using Matlab program.

2. METHODOLOGY

2.1 Bit Error Probability (BER) For Multi-Level Rectangular QAM (MQAM)

In terms of Q-function, the BER for multi-level rectangular QAM (Rct_MQAM) which is denoted as $P_{bRct\_MQAM}(Qfn)$ is given as;

$$P_{bRct\_MQAM}(Qfn) = \left(\frac{4\left(\sqrt{M}-1\right)}{(\mathrm{Log}_2(M))\sqrt{M}}\right) Q\left(\sqrt{\left(\frac{3(\mathrm{Log}_2 M)}{M-1}\right)\left(\frac{\varepsilon_b}{N_0}\right)}\right) \quad (1)$$

A generalized form of the expression for $P_{bRct\_MQAM}(Qfn)$ can be given as;

$$P_{bRct\_MQAM}(Qfn) = \left(A_{Qfn}\right) Q\left(\sqrt{\left(B_{Qfn}\right)\left(\frac{\varepsilon_b}{N_0}\right)}\right) \quad (2)$$

where

$$A_{Qfn} = \frac{4\left(\sqrt{M}-1\right)}{(\mathrm{Log}_2(M))\sqrt{M}} \quad (3)$$

$$B_{Qfn} = \frac{3(\mathrm{Log}_2 M)}{M-1} \quad (4)$$

When complementary error function (erfc), is considered, $P_{bRct\_MQAM}(\mathrm{erfc})$ is given as;

$$P_{bRct\_MQAM}(erfc) = \left(\frac{2\left(\sqrt{M}-1\right)}{(\mathrm{Log}_2(M))\sqrt{M}}\right) erfc\left(\sqrt{\left(\frac{3(\mathrm{Log}_2 M)}{2(M-1)}\right)\left(\frac{\varepsilon_b}{N_0}\right)}\right) \quad (5)$$

A generalized form of the expression for $P_{bRct\_MQAM}(erfc)$ can be given as;

$$P_{bRct\_MQAM}(erfc) = (A_{erfc}) erfc\left(\sqrt{(B_{erfc})\left(\frac{\varepsilon_b}{N_0}\right)}\right) \quad (6)$$

where

$$A_{erfc} = \frac{2(\sqrt{M}-1)}{(\text{Log}_2(M))\sqrt{M}} \quad (7)$$

$$B_{erfc} = \frac{3(\text{Log}_2 M)}{2(M-1)} \quad (8)$$

Similarly, when error function (erfc), is considered, $P_{bRct\_MQAM}(erf)$ is given as;

$$P_{bRct\_MQAM}(erf) = \left(\frac{2(\sqrt{M}-1)}{(\text{Log}_2(M))\sqrt{M}}\right)\left(1 - erf\left(\sqrt{\left(\frac{3(\text{Log}_2 M)}{2(M-1)}\right)\left(\frac{\varepsilon_b}{N_0}\right)}\right)\right) \quad (9)$$

2.2 Symbol Error Probability (SER) For multi-level rectangular QAM (MQAM)

In terms of Q-function, $P_{sRct\_MQAM}(Qfn)$ which is the SER for multi-level rectangular QAM is given as;

$$P_{sRct\_MQAM}(Qfn) = \left(\frac{4(\sqrt{M}-1)}{\sqrt{M}}\right) Q\left(\sqrt{\left(\frac{3}{M-1}\right)\left(\frac{\varepsilon_s}{N_0}\right)}\right) \quad (10)$$

When complementary error function (erfc), is considered, $P_{sRct\_MQAM}(erfc)$ is given as;

$$P_{sRct\_MQAM}(erfc) = \left(\frac{2(\sqrt{M}-1)}{\sqrt{M}}\right) erfc\left(\sqrt{\left(\frac{3}{2(M-1)}\right)\left(\frac{\varepsilon_s}{N_0}\right)}\right) \quad (11)$$

Similarly, when error function (erf), is considered, $P_{sRct\_MQAM}(erf)$ is given as;

$$P_{sRct\_MQAM}(erf) = \left(\frac{2(\sqrt{M}-1)}{\sqrt{M}}\right)\left(1 - erf\left(\sqrt{\left(\frac{3}{2(M-1)}\right)\left(\frac{\varepsilon_s}{N_0}\right)}\right)\right) \quad (12)$$

3.0 RESULTS AND DISCUSSION

The computation of the BER for the multi-level rectangular QAM modulation scheme was performed in Matlab for modulation order ranging from 4 to 4096 and the values of the coefficients, A_{Qfn} and B_{Qfn} as well as A_{erfc} and B_{erfc} were obtained , as shown in Table 1 and Table 2 respectively. Notably, at M =16, $A_{Qfn} = 3/4$ and $B_{Qfn} = 4/5$, $A_{erfc} = 3/8$ and B_{erfc} =2/5.

The results on the BER versus Eb/No for various modulation order of the Multi-Level Rectangular QAM are given in Table 3 and Figure 1. Similarly, the results of BER versus modulation order of the multi-level rectangular QAM for various Eb/No are shown in Table 4 and Figure 2. The results in Table 3 , Figure 2, Table 4 and Figure 2 show that, at Eb/No= 4 dB; BER = 1.2501E-02 for M = 4, BER = 5.8618E-02 for M = 16, 1.1576E-01 for M = 64, BER = 1.4691E-01 for M = 256, and BER = 1.5230E-01 for M = 1024. Essentially, the BER is lowest at M= 4 and highest at M =1024. In all, the BER increases with M for any given Eb/No.

Table 1 Values of the parameters A_{Qfn} and B_{Qfn} for Q-function-based BER for the Multi-Level Rectangular QAM Modulation scheme

M	A_{Qfn}	B_{Qfn}	$P_{bRct\_MQAM}(Qfn)$
4	1	2	$(1)Q\left(\sqrt{(2)\left(\frac{\varepsilon_b}{N_0}\right)}\right)$
16	3/4	4/5	$(3/4)Q\left(\sqrt{(4/5)\left(\frac{\varepsilon_b}{N_0}\right)}\right)$
64	7/12	2/7	$(7/12)Q\left(\sqrt{(2/7)\left(\frac{\varepsilon_b}{N_0}\right)}\right)$
256	15/32	8/85	$(15/32)Q\left(\sqrt{(8/85)\left(\frac{\varepsilon_b}{N_0}\right)}\right)$
1024	31/80	10/341	$(31/80)Q\left(\sqrt{(10/341)\left(\frac{\varepsilon_b}{N_0}\right)}\right)$
4096	21/64	4/455	$(21/64)Q\left(\sqrt{(4/455)\left(\frac{\varepsilon_b}{N_0}\right)}\right)$

Table 2 Values of the parameters A_{erfc} and B_{erfc} for erfc-function-based BER for the Multi-Level Rectangular QAM Modulation scheme

M	A_{erfc}	B_{erfc}	$P_{bRct\_MQAM}(erfc)$
4	1/2	1	$(1/2)\text{erfc}\left(\sqrt{(1)\left(\frac{\varepsilon_b}{N_0}\right)}\right)$
16	3/8	2/5	$(3/8)\text{erfc}\left(\sqrt{(2/5)\left(\frac{\varepsilon_b}{N_0}\right)}\right)$
64	7/24	1/7	$(7/24)\text{erfc}\left(\sqrt{(1/7)\left(\frac{\varepsilon_b}{N_0}\right)}\right)$
256	15/64	4/85	$(15/64)\text{erfc}\left(\sqrt{(4/85)\left(\frac{\varepsilon_b}{N_0}\right)}\right)$
1024	31/160	5/341	$(31/160)\text{erfc}\left(\sqrt{(5/341)\left(\frac{\varepsilon_b}{N_0}\right)}\right)$
4096	21/128	2/455	$(21/128)\text{erfc}\left(\sqrt{(2/455)\left(\frac{\varepsilon_b}{N_0}\right)}\right)$

Table 3 BER Versus Eb/No for various modulation order of the Multi-Level Rectangular QAM

Signal Levels or Modulation Order, M	4	16	64	256	1024
K bits/symbol	2	4	6	8	10
Eb/No(dB)	**Rct_MQAM BER For M=4**	**Rct_MQAM BER For M=16**	**Rct_MQAM BER For M=64**	**Rct_MQAM BER For M=256**	**Rct_MQAM BER For M=1024**
0	7.8650E-02	1.3916E-01	1.7295E-01	1.7789E-01	1.6741E-01
2	3.7506E-02	9.7559E-02	1.4612E-01	1.6391E-01	1.6068E-01
4	1.2501E-02	5.8618E-02	1.1576E-01	1.4691E-01	1.5230E-01
6	2.3883E-03	2.7871E-02	8.3473E-02	1.2667E-01	1.4194E-01
8	1.9091E-04	9.2472E-03	5.2320E-02	1.0334E-01	1.2925E-01
10	3.8721E-06	1.7542E-03	2.6533E-02	7.7807E-02	1.1395E-01
12	9.0060E-09	1.3866E-04	9.7240E-03	5.2022E-02	9.5984E-02
14	6.8101E-13	2.7632E-06	2.1540E-03	2.9098E-02	7.5707E-02
16	0.0000E+00	6.2502E-09	2.1717E-04	1.2400E-02	5.4235E-02
18	0.0000E+00	4.5222E-13	6.3511E-06	3.4721E-03	3.3663E-02
20	0.0000E+00	0.0000E+00	2.6339E-08	5.0531E-04	1.6819E-02
22	0.0000E+00	0.0000E+00	4.9744E-12	2.6336E-05	6.0244E-03
24	0.0000E+00	0.0000E+00	0.0000E+00	2.7204E-07	1.2877E-03

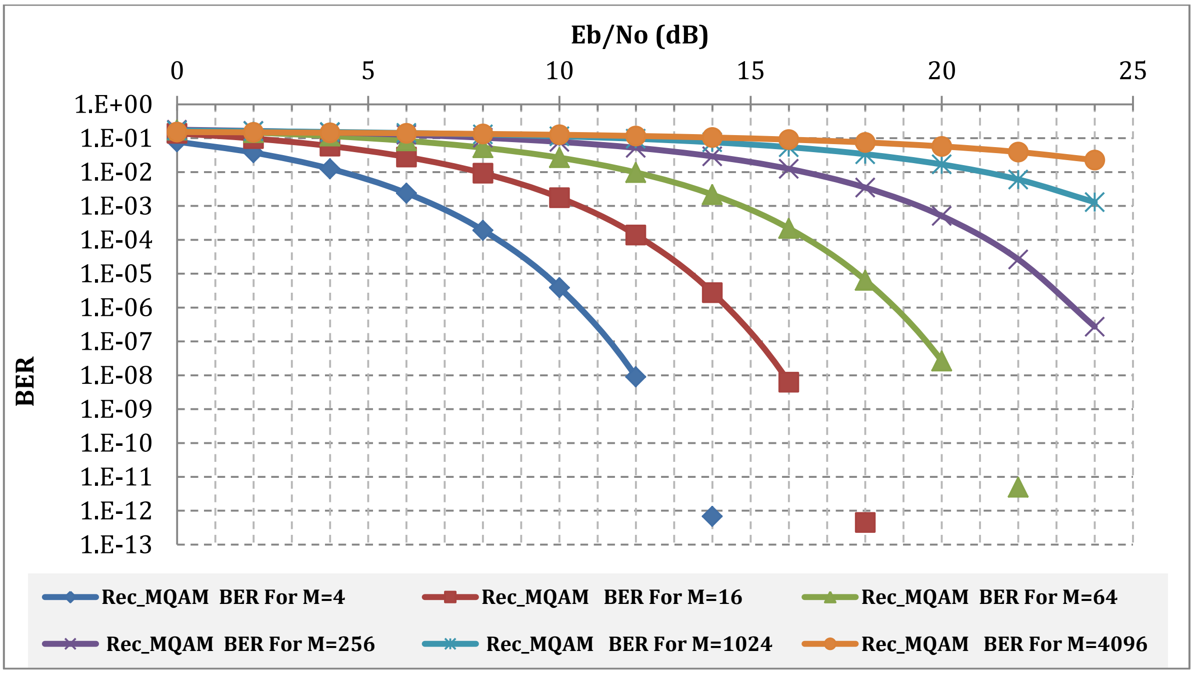

Figure 1 BER Versus Eb/No for various modulation order of the Multi-Level Rectangular QAM

Table 4 BER Versus Modulation Order of the Multi-Level Rectangular QAM for various Eb/No

Signal Levels or Modulation Order, M	BER for Eb/No = 4 dB	BER for Eb/No = 8 dB	BER for Eb/No = 12 dB
4	**1.2501E-02**	**1.9091E-04**	**9.0060E-09**
16	5.8618E-02	9.2472E-03	1.3866E-04
64	1.1576E-01	5.2320E-02	9.7240E-03
256	1.4691E-01	1.0334E-01	5.2022E-02

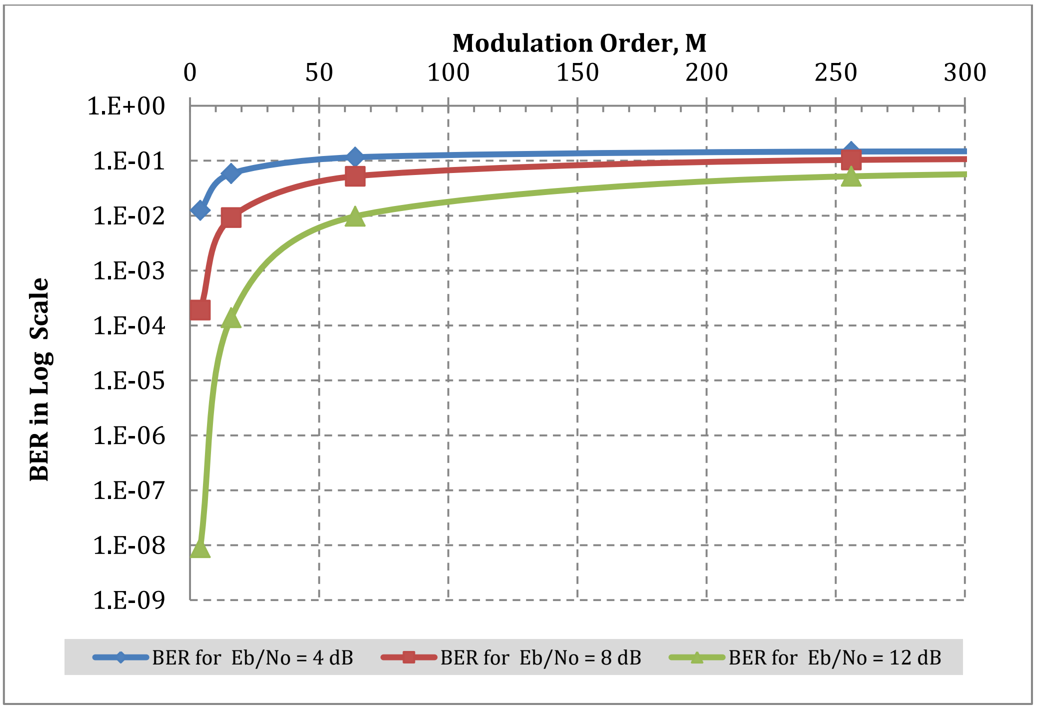

Figure 2 BER Versus Modulation Order of the Multi-Level Rectangular QAM for various Eb/No

4.0 CONCLUSION

Multi-Level Rectangular QAM (MQAM) modulation scheme is studied. The bit error rate (BER) is computed using analytical models that are presented in the paper. Matlab program was used to implement the computation. The BER was computed for various modulation order and Eb/No . The results showed that the BER increases with modulation order for any given Eb/No.

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