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# STRICT COMPLIMENTARY ROOT-BASED ITERATIVE SOLUTION TO THE KEPLER'S EQUATION FOR SATELLITE WITH ELLIPTICAL OR CIRCULAR ORBITS

**Israel Sylvester Umana**[1]
Dept. of Electrical/Electronic & Computer Engineering, University of Uyo, Nigeria

**Akpasam Joseph Ekanem**[2]
Department of Electrical and Electronic Engineering,
Akwa Ibom State University Mkpat Enin, Akwa Ibom State, Nigeria

**Ogungbemi Emmanuel Oluropo**[3]
Dept. of Electrical/Electronic & Computer Engineering, University of Uyo, Nigeria

*Abstract*— In this paper, strict complimentary root-based iterative solution to the Kepler's equation for satellite with elliptical or circular orbits is presented. Specifically, the approach is used to determine the eccentricity anomaly (E) of the orbit for a given set of values of orbital eccentricity (e) and mean anomaly (M). The performance of the Strict Complementary Root Seeded Secant (SCRSS) Method in computing the eccentricity anomaly of the elliptical or circular orbits is compared with that of fixed point iteration (FPI) method. Both the SCRSS method and the FPI method utilised a single initial guess value of E which in this paper is estimated using a piecewise function which specified the initial guess values for three different range of values of M. The computation of E was implemented with Matlab software. The results of the computation for e = 0.095 and M =15° with error tolerance of $\varepsilon \leq 10^{-15}$ show that the FPI converged after 11 cycles whereas the SCRSS converged after 3 cycles. Similarly, the results of the computation for e = 0.095 and M =45° show that the FPI converged after 10 cycles whereas the SCRSS converged after 3 cycles. Furthermore, the results of the computation of E for e = 0.095 and values of M ranging from 5° to 75° show that the convergence cycle for the SCRSS algorithm is 3 while that of the FPI algorithm varied from 6 at M =75° to 13 at M = 5°. Again, the results of the computation of E for e = 0.995 and values of M ranging from 5° to 75° show that the convergence cycle for the SCRSS algorithm is 4 while that of the FPI algorithm varied from 50 at M =75° to 71 at M = 5°. In all, the SCRSS algorithm converges much faster than the FPI algorithm.

***Keywords— Strict Complimentary Root-Based Iterative , Eccentricity Anomaly , Kepler's Equation , Eccentricity , Satellite Elliptical Orbits, Fixed Point Iteration, Mean Anomaly***

## 1. Introduction

Over the years, solution of Kepler's equation applicable to different types of satellite orbits has attracted much researches [1,2,3,4,5,6,7,8,9,10]. Mostly, iterative solution approaches are adopted due to the transcendental nature of the equation when the desired parameter is the eccentricity anomaly (E) as a function of the orbit eccentricity (e) and orbit mean anomaly (M) [11,12,13,14,15,16,17,18,19,20]. Again, the fixed point iteration (FPI) has also been widely adopted for this equation due to the FPI method and the fact that only a single initial value is required to perform the FPI algorithm.

However, in many case, the convergence of the FPI method is very poor, hence it requires several iterations before the desired value of E with the requisite accuracy level is attained [21,22,23,24,25,26]. As such, researchers has resorted to other seeded iteration methods that requires only single initial value for E but with much better convergence performance than the FPI method. According, in this paper, a Strict Complementary Root Seeded Secant (SCRSS) Method is presented and compared with the FPI method [27,28,29]. The SCRSS method requires a single initial guess value for E but converges faster than the FPI method.

Furthermore, in order to further improve on the convergence performance of the iteration methods, a good initial guess value of E is required. As such, in this paper, a piecewise function which specified the initial guess values of E for three different range of values of M is adopted. Finally, the convergence performance of the FPI method and that of the SCRSS method are determined and compared using some numerical examples.

## 2. Methodology

The Kepler's equation for eccentricity anomaly, E is expressed in terms of the orbital eccentricity and the mean anomaly as follows;

$$E = M + e(Sin(E)) \qquad (1)$$

In order to solve the transcendental Kepler's equation for E, iterative solution is usually adopted. In this paper, three different iterative approaches are presented, each with the initial guess value of E denoted as $E_0$ and as expressed by [30], where;

$$E_0 = \begin{cases} M + \left((6M)^{1/3} - M\right)e^2 & for\ 0 \leq M \leq 0.25 \\ M + e\left(\frac{\sin(M)}{1 - Sin(M+e) + Sin(M)}\right) & for\ 0.25 \leq M \leq 2 \\ M + e\left(\frac{e(\sin(M))}{\sqrt{(1 - 2eCos(M) + e^2)}}\right) & for\ 2 \leq M \leq \pi \end{cases} \qquad (2)$$

The subsequent values of E are determined through iterative approach with a termination error tolerance, ∈. The three iterative solutions considered in this paper are: classical secant method, onetime complementary root seeded secant method and strict complementary root seeded secant method.

### 2.1 The Fixed Point Method

For the fixed point method, one initial guess value of E is required. Let the single initial guess value of E be denoted as $E_0$ where its value is as define by [30]. Hence, the analytical procedure for defining the fixed iteration method is given as follows;

At the initial cycle, x = 0, then;

$$E_x = E_0 = \begin{cases} M + \left((6M)^{1/3} - M\right)e^2 & for\ 0 \le M \le 0.25 \\ M + e\left(\frac{\sin(M)}{1 - Sin(M+e) + Sin(M)}\right) & for\ 0.25 \le M \le 2 \\ M + e\left(\frac{e(\sin(M))}{\sqrt{(1 - 2eCos(M) + e^2)}}\right) & for\ 2 \le M \le \pi \end{cases} \quad (3)$$

$$E_{x+1} = M + e(Sin(E_x)) \quad (4)$$

$$f(E_x) = E_x - E_{x+1} \quad (5)$$

If $f(E_x) \le \in$ then stop otherwise x =x +1 and then repeat the cycle by calculating $E_{x+1}$ and $f(E_x)$ and checking the termination condition $f(E_x) \le \in$. The fixed iteration algorithm for the computation of E based on the given initial value of $E_0$ and estimation error, ∈ is as follows;

Step 1: Input M, e , ∈

Step 2 : x=0

Step 3 :

$$E_x = \begin{cases} M + \left((6M)^{1/3} - M\right)e^2 & for\ 0 \le M \le 0.25 \\ M + e\left(\frac{\sin(M)}{1 - Sin(M+e) + Sin(M)}\right) & for\ 0.25 \le M \le 2 \\ M + e\left(\frac{e(\sin(M))}{\sqrt{(1 - 2eCos(M) + e^2)}}\right) & for\ 2 \le M \le \pi \end{cases}$$

Step 4 : $E_{x+1} = M + e\left(Sin(E_x)\right)$

Step 5 : $f(E_x) = E_x - \left(M + e(Sin(E_x))\right)$

Step 6 : If $f(E_x) \le \in$ then goto step 8 else x = x +1; goto step 4

Step 8 : Output $E_{x+1}$

Step 9 : End

## 2.2 Strict Complementary Root Seeded Secant Method

The strict complementary root seeded secant method is defined as follows; at x=0

$$E_0 = \begin{cases} M + \left((6M)^{1/3} - M\right)e^2 & for\ 0 \le M \le 0.25 \\ M + e\left(\frac{\sin(M)}{1 - Sin(M+e) + Sin(M)}\right) & for\ 0.25 \le M \le 2 \\ M + e\left(\frac{e(\sin(M))}{\sqrt{(1 - 2eCos(M) + e^2)}}\right) & for\ 2 \le M \le \pi \end{cases} \quad (3)$$

$$E_{x+1} = M + e(Sin(E_x)) \quad (4)$$

$$f(E_x) = E_x - E_{x+1} \quad (5)$$

$$f(E_{x+1}) = E_{x+1} - \left(M + e(Sin(E_{x+1}))\right) \quad (6)$$

$$E_{x+2} = \frac{E_x\left(f(E_{x+1})\right) - E_{x+1}\left(f(E_x)\right)}{f(E_{x+1}) - f(E_x)} \quad (7)$$

$$f(E_{x+2}) = E_{x+2} - \left(M + e(Sin(E_{x+2}))\right) \quad (8)$$

If $f(E_{x+2}) \le \in$ then stop otherwise x =x +1 , $E_x = E_{x+1}$ and then repeat the cycle by calculating $E_{x+1}$, $f(E_x)$ , $f(E_{x+1})$, $E_{x+2}$, $f(E_{x+2})$ and checking the termination condition $f(E_{x+2}) \le \in$. The strict complementary root seeded secant algorithm for the computation of E based on the given single initial value of $E_x$ and estimation error, ∈ is as follows;

Step 1: Input M, e , ∈

Step 2 : x=0

Step 3 :

$$E_0 = \begin{cases} M + \left((6M)^{1/3} - M\right)e^2 & for\ 0 \le M \le 0.25 \\ M + e\left(\frac{\sin(M)}{1 - Sin(M+e) + Sin(M)}\right) & for\ 0.25 \le M \le 2 \\ M + e\left(\frac{e(\sin(M))}{\sqrt{(1 - 2eCos(M) + e^2)}}\right) & for\ 2 \le M \le \pi \end{cases}$$

Step 4 :

$$E_{x+1} = M + e(Sin(E_x))$$

Step 5 : $f(E_x) = E_x - E_{x+1}$

Step 6 : $f(E_{x+1}) = E_{x+1} - \left(M + e(Sin(E_{x+1}))\right)$

Step 7 : $E_{x+2} = \frac{E_x\left(f(E_{x+1})\right) - E_{x+1}\left(f(E_x)\right)}{f(E_{x+1}) - f(E_x)}$

Step 8 :

$$f(E_{x+2}) = E_{x+2} - \left(M + e(Sin(E_{x+2}))\right)$$

Step 9 :

If $f(E_{x+2}) \le \in$ then goto step 10 else x = x +1; $E_x = E_{x+1}$ goto step 4

Step 10 : Output $E_{x+2}$

Step 11 : End

## 3. RESULTS AND DISCUSSION

The computation of E based on Kepler's equation was implemented with Matlab software. The results of the computation using the fixed point iteration (FPI) method for e = 0.095 and M =15° with error tolerance of $\varepsilon \le 10^{-15}$ are presented in Table 1 while Table 2 is the result of Strict Complementary Root Seeded Secant (SCRSS) Method for e = 0.095 and M =15° with error tolerance of $\varepsilon \le 10^{-15}$ . The results show that the FPI converged after 11 cycles whereas the SCRSS converged after 3 cycles.

Similarly, the results of the computation using the fixed point iteration (FPI) method for e = 0.095 and M =45° with error tolerance of $\varepsilon \le 10^{-15}$ are presented in Table 3 while Table 4 is the result of Strict Complementary Root Seeded Secant (SCRSS) Method for e = 0.095 and M =45° with error tolerance of $\varepsilon \le 10^{-15}$ . The results show that the FPI converged after 10 cycles whereas the SCRSS converged after 3 cycles.

The convergence performance of the two algorithms is demonstrated be computing the value of E for a given e and various values of M. Specifically, the results of the computation of E for e = 0.095 and values of M ranging from 5° to 75° are given in Table 5 and Figure 1. The results show that for the given range of values of M, the convergence cycle for the SCRSS algorithm is 3 while that of the FPI algorithm varied from 6 at M =75° to 13 at M = 5°.

Again, the results of the computation of E for e = 0.995 and values of M ranging from 5° to 75° are given in Table 6 and Figure 2. The results show that for the given range of values of M, the convergence cycle for the SCRSS algorithm is 4 while that of the FPI algorithm varied from 50 at M =75° to 71 at M = 5°. In all, the SCRSS algorithm converges much faster than the FPI algorithm.

Table 1 Result of the fixed point iteration (FPI) method for e = 0.095 and M =15° with error tolerance of $\varepsilon \leq 10^{-15}$

| **E = 0.095** | **M =15°** | **M =0.261799 radians** | |
|---|---|---|---|
| | **Eo =16.54888682°** | **Eo = 0.288833 radians** | |
| **Cycle x** | **Ex** | **f(Ex)** | **Ex+1** |
| 1 | 2.888326E-01 | -2.60E-05 | 2.888586E-01 |
| 2 | 2.888586E-01 | -2.37E-06 | 2.888609E-01 |
| 3 | 2.888609E-01 | -2.16E-07 | 2.888611E-01 |
| 4 | 2.888611E-01 | -1.96E-08 | 2.888612E-01 |
| 5 | 2.888612E-01 | -1.79E-09 | 2.888612E-01 |
| 6 | 2.888612E-01 | -1.63E-10 | 2.888612E-01 |
| 7 | 2.888612E-01 | -1.48E-11 | 2.888612E-01 |
| 8 | 2.888612E-01 | -1.35E-12 | 2.888612E-01 |
| 9 | 2.888612E-01 | -1.23E-13 | 2.888612E-01 |
| 10 | 2.888612E-01 | -1.12E-14 | 2.888612E-01 |
| 11 | 2.888612E-01 | -9.99E-16 | 2.888612E-01 |
| 12 | 2.888612E-01 | 0.00E+00 | 2.888612E-01 |

Table 2 Result of the Strict Complementary Root Seeded Secant (SCRSS) Method for e = 0.095 and M =15°

| E = 0.095 | M = 15° | M = 0.261799 radians | Eo =16.54888682° | Eo = 0.288833 radians | | |
|---|---|---|---|---|---|---|
| Cycle x | Ex | f(Ex) | Ex+1 | f(Ex+1) | Ex+2 | f(Ex+2) |
| 1 | 2.888326E-01 | -2.60E-05 | 2.888586E-01 | 2.89E-01 | 2.888612E-01 | 1.01E-12 |
| 2 | 2.888612E-01 | 1.01E-12 | 2.888612E-01 | 2.89E-01 | 2.888612E-01 | 0.00E+00 |
| 3 | 2.888612E-01 | 0.00E+00 | 2.888612E-01 | 2.89E-01 | | |

Table 3 Result of the fixed point iteration (FPI) method for e = 0.095 and M = 45°

| E = 0.095 | M = 45° | M = 0.785398radians | |
|---|---|---|---|
| | Eo =49.11151967 ° | Eo = 0.857158 radians | |
| Cycle x | Ex | f(Ex) | Ex+1 |
| 1 | 8.571577E-01 | -5.90E-05 | 8.572167E-01 |
| 2 | 8.572167E-01 | -3.67E-06 | 8.572204E-01 |
| 3 | 8.572204E-01 | -2.28E-07 | 8.572206E-01 |
| 4 | 8.572206E-01 | -1.42E-08 | 8.572207E-01 |
| 5 | 8.572207E-01 | -8.83E-10 | 8.572207E-01 |
| 6 | 8.572207E-01 | -5.49E-11 | 8.572207E-01 |
| 7 | 8.572207E-01 | -3.41E-12 | 8.572207E-01 |
| 8 | 8.572207E-01 | -2.12E-13 | 8.572207E-01 |
| 9 | 8.572207E-01 | -1.31E-14 | 8.572207E-01 |
| 10 | 8.572207E-01 | -8.88E-16 | 8.572207E-01 |
| 11 | 8.572207E-01 | 0.00E+00 | 8.572207E-01 |

Table 4 Result of the Strict Complementary Root Seeded Secant (SCRSS) Method for e = 0.095 and M =45°

| E = 0.095 | M = 45° | M =0.785398 radians | Eo = 49.11151967 ° | Eo = 0.857158 radians | | |
|---|---|---|---|---|---|---|
| Cycle x | Ex | f(Ex) | Ex+1 | f(Ex+1) | Ex+2 | f(Ex+2) |
| 1 | 8.571577E-01 | -5.90E-05 | 8.572167E-01 | 8.57E-01 | 8.572207E-01 | 8.85E-12 |
| 2 | 8.572207E-01 | 8.85E-12 | 8.572207E-01 | 8.57E-01 | 8.572207E-01 | 0.00E+00 |
| 3 | 8.572207E-01 | 0.00E+00 | 8.572207E-01 | 8.57E-01 | #DIV/0! | #DIV/0! |

Table 5 Convergence Cycle (n) for the cases where e = 0.095

| e | M in degrees | Eo in degree | Ein degrees | Convergence Cycle (n) for FPI Method | Convergence Cycle (n) for SCRSS Method | P% Reduction in convergence cycle |
|---|---|---|---|---|---|---|
| 0.095 | 5 | 5.371651 | 5.524 | 13 | 3 | 76.9 |
| 0.095 | 15 | 16.54889 | 16.5505 | 11 | 3 | 72.7 |
| 0.095 | 25 | 27.51148 | 27.5146 | 11 | 3 | 72.7 |
| 0.095 | 45 | 49.11152 | 49.1151 | 10 | 3 | 70.0 |
| 0.095 | 55 | 59.69693 | 59.6995 | 9 | 3 | 66.7 |
| 0.095 | 75 | 80.366 | 80.3663 | 6 | 3 | 50.0 |
| Average | | | | 10 | 3 | 70.0 |

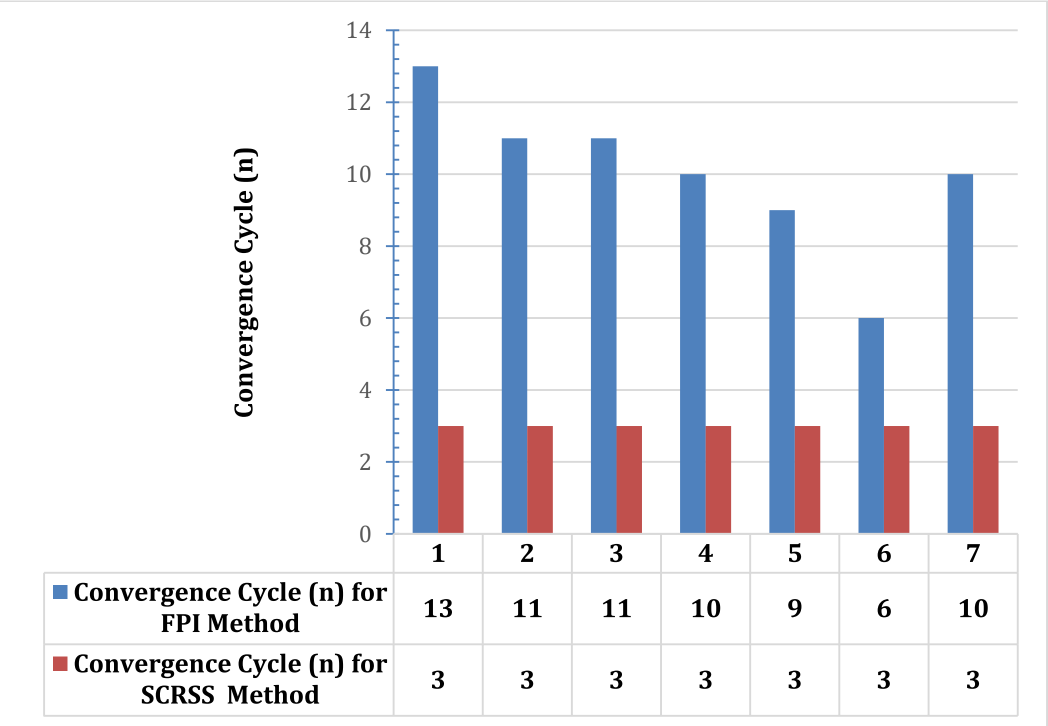


**Figure 1 Convergence Cycle (n) for the cases where e = 0.095**

Table 6 Convergence Cycle (n) for the cases where e = 0.995

| e | M in degrees | Eo in degree | Ein degrees | Convergence Cycle (n) for FPI Method | Convergence Cycle (n) for SCRSS Method | P% Reduction in convergence cycle |
|---|---|---|---|---|---|---|
| 0.995 | 5 | 45.7694 | 46.0293 | 71 | 4 | 94.4 |
| 0.995 | 15 | 62.95092 | 67.7731 | 32 | 5 | 84.4 |
| 0.995 | 25 | 80.72897 | 81.3628 | 16 | 4 | 75.0 |
| 0.995 | 45 | 100.2977 | 100.968 | 18 | 4 | 77.8 |
| 0.995 | 55 | 107.3518 | 108.927 | 27 | 4 | 85.2 |
| 0.995 | 75 | 120.03 | 122.878 | 50 | 4 | 92.0 |
| Average | | | | 35.7 | 4.2 | 88.3 |

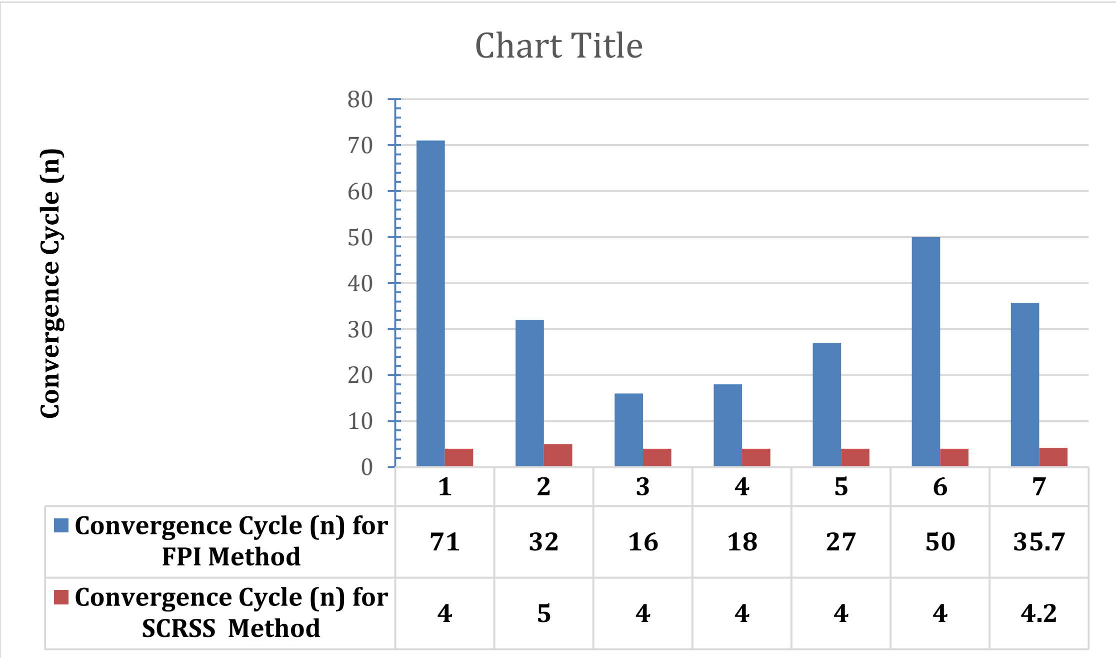


Figure 2 Convergence Cycle (n) for the cases where e = 0.995

## 4.0 CONCLUSION

A seeded secant approach for computing the eccentricity anomaly for satellites with elliptical or circular orbits is presented. The approach used complementary-root form of the Kepler's equation in similar way are is applicable to the popular fixed iteration method. Hence, the convergence performance of the Strict Complementary Root Seeded Secant (SCRSS) Method is compared with that of the fixed point iteration (FPI) method. In all, the results showed that in the computation of the eccentricity anomaly for elliptical and circular orbits, the SCRSS method converged much faster than the FPI method in all the cases considered.

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