

# Performance Analysis of MIMO Diversity Schemes in LTE Networks

Ibe I. Igajah<sup>1</sup>, Kufre M. Udofia<sup>2</sup>, Kingsley M. Udofia<sup>3</sup>

<sup>1,2,3</sup> Department of Electrical/Electronic and Computer Engineering, University of Uyo,  
Uyo, Nigeria

(<sup>1</sup>igajah55@gmail.com, <sup>2</sup>kufremudofia@uniuyo.edu.ng, <sup>3</sup>Kingsleyudofia@uniuyo.edu.ng)

**Abstract—** In modern wireless communication devices, high data transfer rate is essential for multimedia services desirably with low latency. In this paper, Bit Error Rate (BER) analysis of diversity scheme with Single User (SU) and Multi-User (MU) MIMO in the 3GPP Long Term Evolution (LTE) system has been evaluated. Analytical expressions for the average BER of the system was derived over flat Rayleigh fading channels for two different MIMO schemes (SFBC and FSTD diversity schemes) as defined in LTE Rel. 8. M-ary quadrature amplitude modulation (M-QAM) schemes was assumed for easy analysis and numerical evaluation. The derived analytical capacity expressions for SFBC- and FSTD-OFDM diversity schemes were simulated against theoretical Monte-Carlo channel capacity. At seemingly low SNR, very insignificant variation in capacity between the different schemes was observed with the disparity becoming more obvious as SNR increased. At SNR of 31 dB, an improvement of 1.24 bps/Hz and 1.78 bps/Hz were achieved by the 2 x 1 SFBC-OFDM scheme and 2 x 2 FSTD-OFDM scheme, respectively. With regard to the 4 x 2 FSTD-OFDM scheme, higher margin of improvement (3.05 bps/Hz) was observed in comparison to SISO scheme. Also, results from derived analytical equations from capacity analysis show close alignment to the results from Monte-Carlo simulation, which confirms the exactness of the mathematical expressions derived.

**Keywords—** Bit Error Rate; Single User (SU) MIMO; Multi-User (MU) MIMO; Monte-Carlo; Rayleigh fading; SFBC- and FSTD-OFDM.

## I. INTRODUCTION

The enormous gains of innovations in mobile wireless broadband services encouraged users who ordinarily would prefer to use other means of communication to adopt wireless broadband communication system thereby leading to increased subscription of network resources especially considering the limited bandwidth with consequent traffic congestion during peak hours [1]. In order to minimize this congestion, Long Term Evolution (LTE) mobile communication standard was introduced to take advantage of Orthogonal Frequency Division Multiplexing (OFDM) and multiple antenna transmission and reception that became popularly known as Multiple Input Multiple Output (MIMO).

MIMO technology is considered a breakthrough in wireless communication used for exploiting multipath propagation [2]. MIMO techniques deliver substantial performance improvements in terms of data transmission rate and interference reduction. It is generally used to increase data rates through multiplexing and diversity schemes [3]. Among the aims of MIMO adoption are improved spectral efficiency, lower inter-symbol-interference (ISI) and interference from other users can be minimized using smart antenna techniques. For example, traditional Wi-Fi router with Single User MIMO (SU-MIMO) technology can stream data to only one device at a time; when multiple devices are connected, each device needs to take its turn and wait before receiving more data which leads to slower network and internet speed. On the other hand, [4] reported that using Multi-User MIMO (MU-MIMO) technology, a network Access Point (AP) can stream data simultaneously to connected devices, this increase the network throughput by more than a factor of three which is representative of faster download and connectivity speed. According to [5], MIMO can be used in two basic ways: multiplexing and diversity. When MIMO is used for diversity, the aim is basically to increase network reliability while throughput enhancement is usually the target when MIMO multiplexing is the intention.

In this paper, the focus is on performance analysis of MIMO diversity schemes implemented on LTE network done with the aim of increasing the reliability of the network through the use of multi-antenna system that enables both transmit and receive diversity.

## II. RESEARCH METHODOLOGY

Fig. 1 gives an illustration of the research methodology. Details of the various tiers captured in the research methodology are outlined in the following subsections.

### A. BER of LTE MIMO Schemes - SFBC

The two main diversity schemes - Space Frequency Block Code (SFBC) and Frequency Shift Time Diversity (FSTD) – applicable to LTE systems adopt 2 and 4 transmit antennas, respectively alongside a single data stream [6]. For transmit diversity operation with two antennas integrated at the eNodeB, SFBC is utilized. Application of Alamouti for two antennas on Space Time Block Codes (STBC) yields SFBC [7]. Being that the subframe of OFDM

symbols in LTE is a mixture of odd, and even numbers, and the two-dimensional nature of LTE signal (frequency and time domain), complication arises from direct application of STBC in LTE as is obtainable in UMTS standard. Hence, the transmission (for SFBC) of the symbols in LTE is done from two antenna ports at the eNodeB on adjacent subcarrier pairs as illustrated in (1) [8].

$$\begin{bmatrix} y^{(0)}(1) & y^{(0)}(2) \\ y^{(1)}(1) & y^{(1)}(2) \end{bmatrix} = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix} \quad (1)$$

where  $y^{(n)}(q)$  represents the transmitted symbols on the  $q^{th}$  subcarrier from  $n$  – antenna port. The purpose of negation and conjugation of the transmit signal vector,  $x_i$  is to ensure that the signal remain uncorrelated. It is required that the transmitted signal stream remain orthogonal so that an optimal performance can be derived from simple linear receivers when adopted. Bearing in mind that OFDM converts the channels from multiple paths to Q frequency-flat fading channel, it is expedient to first derive BER expression based on Rayleigh fading channels denoted as  $P_b(E)$  assuming the effect of cyclic prefix in OFDM to be negligible. Thus, (2) is used to compute the average BER over Q-subcarriers.

$$BER_{avg} = \frac{1}{Q} \sum_{q=1}^Q P_{b,q}(E) \quad (2)$$

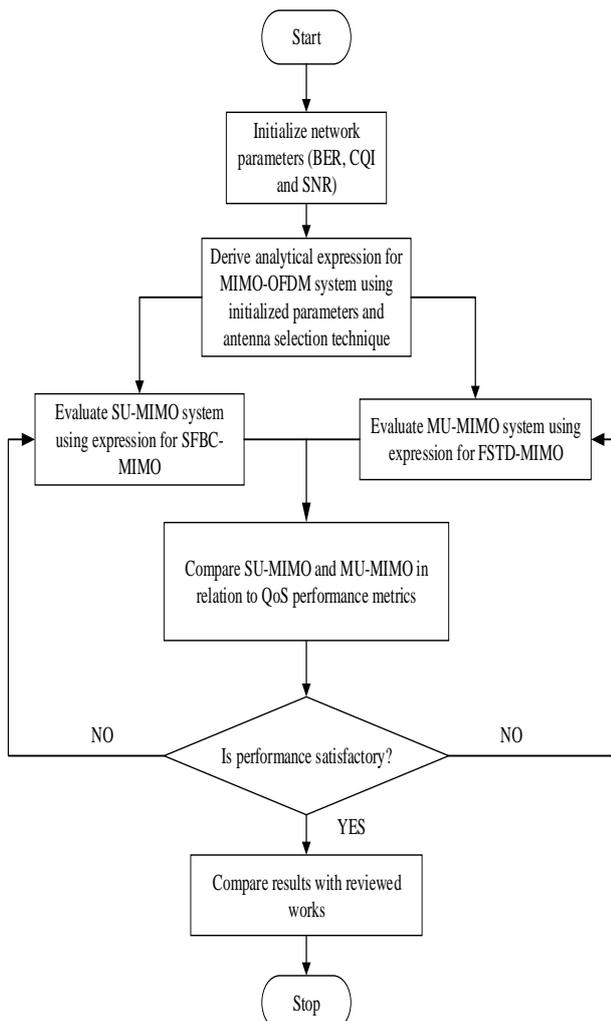


Fig. 1 Research methodology

From the Chi-squared distribution function, the probability density function (PDF) in relation to SNR is given for  $2 \times 1$  SFBC-MIMO scheme in (3) [9].

$$f(\gamma) = \frac{2}{\bar{\gamma}^2} \gamma e^{-\frac{2}{\bar{\gamma}}\gamma} \quad (3)$$

where  $\bar{\gamma}$  represents the average SNR per symbol ( $\bar{\gamma} = \frac{E_s}{N_0}$ ),  $E_s$  is the average transmit symbol energy per antenna and  $N_0$  is the power spectral density of the AGWN. From (3), BER is evaluated using the method proposed by [10] over generalized fading channel with MGF as a function of  $\bar{\gamma}$  from the PDF of the instantaneous SNR given in (4).

$$M_{\bar{\gamma}}(s) = \int_0^{\infty} e^{-s\gamma} f(\gamma) d\gamma \quad (4)$$

Substituting for  $f(\gamma)$  in Equation 4 and integrating over  $M_{\bar{\gamma}}(s)$  within the given limits while adopting some integration identities yields (5).

$$M_{\bar{\gamma}}(s) = \frac{4}{\bar{\gamma}^2 (s + \frac{2}{\bar{\gamma}})^2} \quad (5)$$

For M-QAM modulation scheme, the average BER expression is as given in (7).

$$P_b(E) \cong B \sum_{i=1}^{\sqrt{M}/2} \frac{1}{\pi} \int_0^{\pi/2} M_{\bar{\gamma}}(A_{i,\theta}) d\theta \quad (7)$$

$$\text{where: } A_{i,\theta} = \frac{(2i-1)^2}{2 \sin^2 \theta} \frac{3}{(M-1)} \text{ and } B = 4 \left( \frac{\sqrt{M}-1}{\sqrt{M}} \right) \left( \frac{1}{\log_2 M} \right)$$

where M is an arbitrary number chosen as an index of selected modulation scheme and  $i$  is an initialization counter. Expressing (6) in terms of  $A_{i,\theta}$  produces (8).

$$M_{\bar{\gamma}}(A_{i,\theta}) = \frac{4}{\bar{\gamma}^2 \left( \left[ \frac{(2i-1)^2}{2 \sin^2 \theta} \frac{3}{(M-1)} \right] + \frac{2}{\bar{\gamma}} \right)^2} \quad (8)$$

By evaluation and simplification, (8) is further reduced to (9).

$$M_{\bar{\gamma}}(A_{i,\theta}) = \left( \frac{\sin^2 \theta}{\sin^2 \theta + v_i} \right)^2 \quad (9)$$

$$\text{where } v_i = \frac{3\bar{\gamma}(2i-1)^2}{2(M-1)} \frac{\bar{\gamma}}{2}$$

substituting (9) into (7) gives (10) which represents the performance indicator for average BER.

$$P_b(E) \cong B \sum_{i=1}^{\sqrt{M}/2} \frac{1}{\pi} \int_0^{\pi/2} \left( \frac{\sin^2 \theta}{\sin^2 \theta + v_i} \right)^2 d\theta \quad (10)$$

The closed-form expression for average BER performance for an M-QAM modulation scheme is derived by analytically solving (10) using the approach from [8] to give (11).

$$P_b(E) \cong B \sum_{i=1}^{\sqrt{M}/2} \tau_2 \left( \frac{\pi}{2}, v_i \right) \quad (11)$$

The closed-form expression for  $\tau_2$  is given by [8], [11] in (12) as follows:

$$\tau_n(\phi, D) = \frac{1}{\pi} \int_0^{\phi} \left( \frac{\sin^2 \theta}{\sin^2 \theta + D} \right)^n d\theta \quad \text{for } -\pi \leq \phi \leq \pi$$

$$= \frac{\beta}{\pi} \left\{ \left( \frac{\pi}{2} + \tan^{-1} \alpha \right) \sum_{q=0}^{n-1} \binom{2q}{q} \frac{1}{[4(1+D)]^q} \dots + \dots \sin(\tan^{-1} \alpha) \sum_{q=1}^{n-1} \sum_{p=1}^q \frac{T_{pq}}{(1+D)^q} [\cos(\tan^{-1} \alpha)]^{2(q-p)+1} \right\} \dots (12)$$

where  $T_{pq} = \binom{2q}{q} \left[ \binom{2(q-p)}{q-p} 4^p [2(q-p) + 1] \right]^{-1}$ ,  $\beta = \sqrt{\frac{D}{1+D}} \operatorname{sgn} \phi$ , and  $\alpha = -\beta \cot \phi$

Similar approach is applied in deriving the expression of average BER of  $4 \times 2$  Frequency Shift Time Diversity (FSTD) OFDM scheme. For 4 transmit antennas, the frequency block code is defined as illustrated in (13).

$$\begin{bmatrix} y^{(0)}(1) & y^{(0)}(2) & y^{(0)}(3) & y^{(0)}(4) \\ y^{(1)}(1) & y^{(1)}(2) & y^{(1)}(3) & y^{(1)}(4) \\ y^{(2)}(1) & y^{(2)}(2) & y^{(2)}(3) & y^{(2)}(4) \\ y^{(3)}(1) & y^{(3)}(2) & y^{(3)}(3) & y^{(3)}(4) \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & 0 & 0 \\ 0 & 0 & x_3 & x_4 \\ -x_2^* & x_1^* & 0 & 0 \\ 0 & 0 & -x_4^* & x_3^* \end{bmatrix} \dots (13)$$

where  $y^{(n)}(q)$  represents the transmitted symbols on the  $q^{th}$  subcarrier from  $n$  – antenna port. The instantaneous SNR of the system in relation to  $4 \times 2$  FSTD MIMO scheme for  $q^{th}$  subcarrier is observed to be equivalent to that of  $2 \times 2$  STBC MIMO. Hence, the PDF of SNR expressed in terms of Chi-square distribution function is given in (14) [9].

$$f(\gamma) = \frac{8}{3\bar{\gamma}^4} \gamma^3 e^{-\frac{2}{\bar{\gamma}} \gamma} \quad (14)$$

The MGF expression in this case is obtained by substituting (14) into (4) to produce (15).

$$M_{\bar{\gamma}}(s) = \frac{16}{\bar{\gamma}^4 \left( s + \frac{2}{\bar{\gamma}} \right)^4} \quad (15)$$

Just as was obtainable in the previous case of SFBC, (15) can be inserted into (7) to give the average BER equation with M-QAM modulation for FSTD in Equation 3.19.

$$P_b(E) \cong B \sum_{i=1}^{\sqrt{M}/2} \frac{1}{\pi} \int_0^{\pi/2} \left( \frac{\sin^2 \theta}{\sin^2 \theta + v_i} \right)^4 d\theta \quad (16)$$

where  $v_i = \frac{3\bar{\gamma}(2i-1)^2 \bar{\gamma}}{2(M-1)}$ , the integral in (16) can be computed numerically or analytically to give the average BER close-form expression as it relates to M-QAM modulation scheme as presented in (17).

$$P_b(E) \cong B \sum_{i=1}^{\sqrt{M}/2} \tau_4 \left( \frac{\pi}{2}, v_i \right) \quad (17)$$

The close-form expression of the term  $\tau_4(\dots)$  can be obtained from (12).

For SISO MIMO systems, the average BER for Raleigh fading channels and M-QAM signals as synthesized from (17) to produce (18) [8].

$$P_b(E) \cong \frac{B}{2} \sum_{i=1}^{\sqrt{M}/2} \left( 1 - \sqrt{\frac{1.5(2i-1)^2 \bar{\gamma} \log_2 M}{M-1+1.5(2i-1)^2 \bar{\gamma} \log_2 M}} \right) \quad (18)$$

### III. SIMULATION, RESULTS AND DISCUSSION

The results of the various analytical expressions derived in the preceding section is presented in this section. Monte-Carlo simulation method was adopted

for the determination of average BER using the Link Level Simulator (Vienna Simulator) with simulation parameters shown in Table 1.

Taking the average BER performance as a function of  $\bar{\gamma} = \frac{E_s}{N_o}$  for Single Input Single Output (SISO),  $2 \times 2$  SFBC and  $4 \times 2$  FSTD MIMO schemes using BPSK and QAM modulation modes, results obtained were plotted and illustrated in Fig. 2 for 16-QAM and Fig. 3 for 64-QAM. A noticeable trend from Fig. 2 and Fig. 3 is a constant decrease by factors of  $\bar{\gamma}^1$ ,  $\bar{\gamma}^2$  and  $\bar{\gamma}^4$  corresponding to SISO,  $2 \times 2$  SFBC and  $4 \times 2$  FSTD cases with the superscripts indicating the corresponding diversity order of 1, 2 and 4. As earlier explained in the previous section, for the  $4 \times 2$  FSTD scheme, for each frequency-slot/time-slot, two out of four available transmit antennas are in use per time thereby giving the equivalence of  $2 \times 2$  diversity order. To avoid repeated simulation and since the corresponding average BER curve for  $4 \times 2$  FSTD is identical to the classical  $2 \times 2$  STBC system with time invariant channel,  $2 \times 2$  specification was omitted from the simulation settings table in Table 1. From the average BER plots given in Fig. 2 to Fig. 3, it is obvious that BER performance improves with corresponding increase in the number of antennas (transmit or receive antennas).

TABLE 1. SIMULATION SETTINGS

Parameters	Settings
Transmission schemes	SISO; 2x1 SFBC; 4x2 FSTD
Bandwidth	5 GHz
Simulation length	2500
Channel type	Flat Raleigh
Channel knowledge	Perfect
CQI	9(16-QAM) and 16(64-QAM)

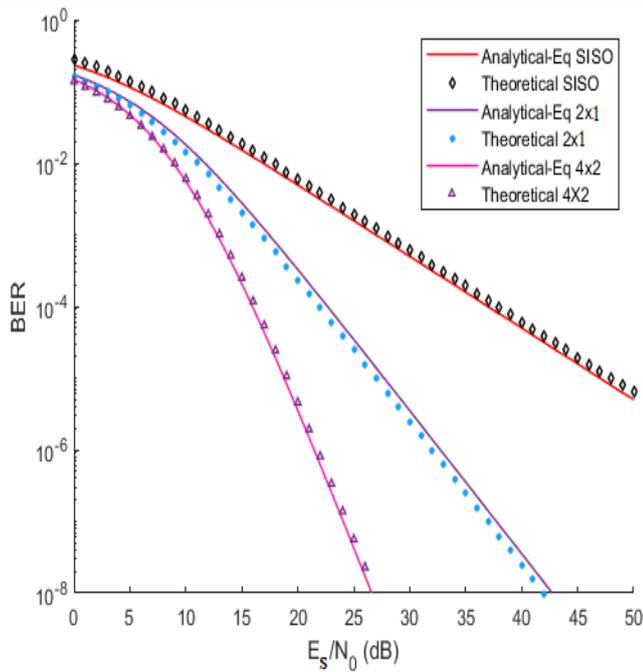


Fig. 2 Average BER analytical expression and Monte-Carlo simulation for 16-QAM modulation

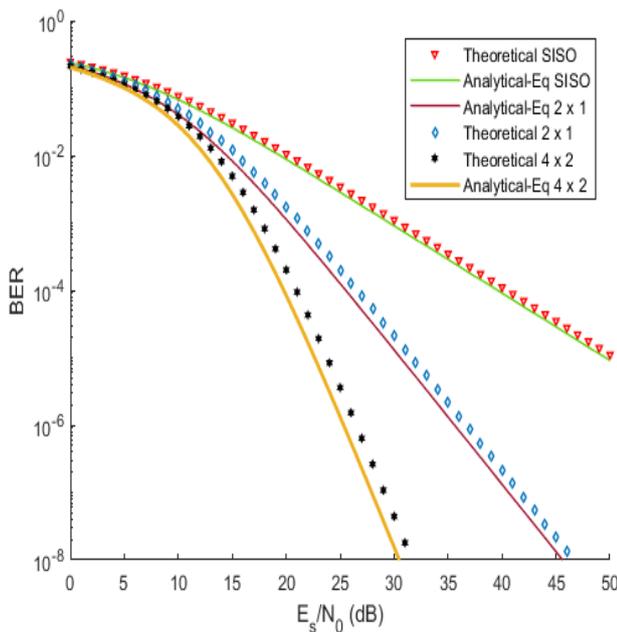


Fig. 3 Average BER analytical expression and Monte-Carlo simulation for 64-QAM Modulation

Precisely, considering Figure 4.3, a BER of  $10^{-5}$  for SISO scheme is achieved with an SNR of 50 dB. Similarly, for the  $2 \times 1$  SFBC scheme, the equivalent level of BER ( $10^{-5}$ ) is achieved at SNR = 28 dB, which represents a 22 dB improvement compared to the SISO scheme. Further improvement is noticeable as the  $4 \times 2$  FSTD scheme performs even more efficiently since an improvement of 28 dB is observed at the same BER level of ( $10^{-5}$ ), as the required SNR is found to be 18 dB. Hence, the SNR gain of the  $4 \times 2$  FSTD scheme compared to the  $2 \times 1$  SFBC scheme is about 10 dB (28 dB - 18 dB). Comparable

observations can be made for the 16-QAM modulation order (Figure 4.4). For this modulation scheme, a BER of  $10^{-4}$  for SISO scheme is obtained with an SNR of approximately 43 dB. For the  $2 \times 1$  SFBC scheme only an SNR of 29 dB is needed to reach the  $10^{-4}$  level of BER, which represents an improvement in SNR of 14 dB. The SNR improvement gain of the  $4 \times 2$  FSTD with respect to SISO scheme is almost 23 dB as the required SNR to achieve  $10^{-4}$  with  $4 \times 2$  FSTD scheme is found to be 20 dB. The SNR gain of  $4 \times 2$  FSTD with respect to  $2 \times 1$  SFBC is reduced to 9 dB (29 dB - 20 dB).

#### IV. CONCLUSION

The analysis and assessment of performance of MIMO-OFDM schemes in LTE systems using BER for two OSTBC diversity MIMO schemes; namely the  $2 \times 1$  SFBC-OFDM and the  $4 \times 2$  FSTD-OFDM in the 3GPP 5 MHz LTE system over a Rayleigh flat fading channel was presented in this paper. The simulation results showed remarkable agreement between numerical results from derived expressions and Monte-Carlo simulation results. For a well-rounded analysis, a study of the channel capacity for  $2 \times 1$  SFBC-OFDM and  $4 \times 2$  FSTD-OFDM MIMO schemes in Rayleigh flat fading channel was also performed. Probability density function of the instantaneous signal to noise ratio was used to derive analytical expressions for the capacity in terms of exponential integral and Poisson distribution.

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